Introduction

The goal of this document is to assist AP Statistics teachers in planning their lessons. Specifically, these lessons will be organized around the textbook *Stats: Modeling the World*. Each post will offer the following:

- A lesson objective
- A few ideas for teaching this lesson, along with pacing suggestions
- Suggested exercises for the lesson (SMW5e)
- Additional resources that support the lesson (AP Classroom, Khan Academy, etc...)
- An exit ticket that can be used to evaluate student learning

A few personal notes are in order. These ideas reflect one person's ideas for how to organize your AP Statistics classroom, thus the title, *Stats My Way*. However, I would be remiss if I did not mention two important things.

For one, I have been helped to an immeasurable degree by others along the way. Dave Bock, Corey Andreasen, Carl Henriksen, Doug Tyson, Adam Yankay, DeAnna MacDonald, and Jeff Eicher are among the few amazing educators who I have learned from (and I could list so many more!). I am deeply indebted to their friendships and to the many things they've taught me. If you find things you like among these materials, they are the ones who deserve the credit.

And finally, there are so many different amazing ways to teach this course. These are some suggestions that have worked for me. Please take them, use them, and most importantly, make them your own. It has been said, "Good teachers borrow, great teachers steal!" I have no doubt that you can take what is here and transform it into something that fits your own amazing teaching style. I trust you will find it helpful.

Enjoy!

Jared Derksen

Unit I—Exploring Data

Lesson 1—Stats Starts Here—CED 1.2

Learning Objectives—Students will learn...

- that statistics is about the collection of data and the study of its variation.
- to describe the individuals that are being measured
- whether the variables being measured are categorical or quantitative.

Lesson suggestions

Create a student survey that the entire class completes. Here is a Google Form to get you started. Another option is to use the Census at Schools survey from the ASA. This survey will take a bit longer, but it can pay dividends later. Stay tuned for more ideas using this site! This lesson is typically my second school day. My class periods are about 55 minutes. (On the first day of school I use the Smelling Parkinson's activity from Doug Tyson.)

Textbook suggestions

Ask students to read Chapter 1. It is well-written and funny! Not only will they learn a lot, they will immediately recognize that reading this text will be enjoyable.
Suggested exercises (SMW 5e): Chapter 1 #7-9, 13-15, 24

Additional resources

AP Classroom Daily Videos

- 1.1—Introducing Statistics
- 1.2—The Language of Variation

Khan Academy practice:

- Individuals, variables, and categorical & quantitative data

Desmos

- Here is a Get-Know-You activity

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 2—Categorical Data, Part 1—CED 1.3 & 1.4

Learning Objectives—Students will learn...

- how to represent categorical data in tables.
- how to represent categorical data in pie and bar graphs.
- the importance of the area principle.
- how to read and interpret categorical tables and graphs, using both frequencies and relative frequencies.

Lesson Suggestions

Using your survey from the previous day can be an engaging way to introduce the lesson.
You can find many poorly done graphs to illustrate how to break the area principle. Here is one list of such graphs from *Business Insider*. This lesson is not very difficult for students, so take advantage of this day to get some of that beginning of the school year stuff completed. I save some of my "go-over the syllabus" reminders for this day. This is my third day of class.

Textbook Suggestions

- Chapter 2 #5, 9, 13-15, 17

Additional Resources

AP Classroom Daily Videos:

- 1.3—Categorical Variables with Tables
- 1.4.1—Categorical Variables with Graphs

Khan Academy Practice:

- Categorical Data Quiz

Theme song: *American Pie*, by Don McLean (pie graphs... 😬). I like to play songs that have a tie-in with the day's lesson. One of the advantages of a zoom classroom is that you have a more captive and quiet audience, so this has been even easier to implement. I often have a song playing while I'm checking in my students.

Video suggestion: This clip from a *How I Met Your Mother* creates a useful mnemonic.

The inventor of the pie graph was William Playfair, who had an amazing life. Florence Nightingale helped popularize the use of pie graphs!

Exit Ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 3—Categorical Data, Part 2—CED 2.2, 2.3, and 2.4

Learning Objectives—Students will learn…

- how to describe contingency tables using marginal and conditional distributions.
- find relative frequencies from two-way tables.
- create and interpret stacked bar graphs and/or mosaic plots.
- determine if two categorical variables appear roughly independent or if there appears to be an association between the variables.

Lesson Suggestions

Pick two anchor examples, one that has a clear association and one with near-perfect independence. I like to use the Titanic data for a clear association. For independence, I usually create fake data for something like gender vs eye color that is scientifically independent. This lesson often takes me two days. Or a full day plus a half day and a quiz (my normal days are 55 minutes, but only 45 minutes on Fridays).

Side note: While the CED includes these topics in Unit 2 (two-variable data), I find it best to leave this topic here. It is a great way to elevate the thinking and communication requirements of the course during the first week of school.

Textbook Suggestions

- Chapter 2 #21, 23, 25, 30, 32, 33

Additional Resources

AP Classroom Daily Videos:

- 1.4.2—Categorical Variables with Graphs
- 2.2—Representing Categorical Variables
- 2.3—Statistics for Two Categorical Variables

Khan Academy Practice:

- Quiz 2 on Categorical Variables
- Mosaic Plots
- Unit Test (cumulative)

There are mosaic plots in the supplementary CED materials that accompany SMW5e. You can find them here. You may want to leave mosaic plots as an investigative task or a team problem-solving activity. I don't take time to have students make one. And it seems that with some critical thinking students can decipher these plots for themselves.

AP Free Response

- 2011 #2 is a question that students can tackle this time of year. While it asks for "probability" instead of "percent", students can generally figure out parts (a) and (b). I like to have students work together in groups to solve part (c).
- 2014 #1ab is a more straightforward option. Just don't assign part (c)!

Here is my categorical project. It requires students to use StatCrunch.com. You can modify the project by having students use a free stats website to compute the statistics and then pasting their results into a document.

Exit Ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 4—Introduction to Describing/Comparing Quantitative Data—CED1.5 & 1.6

Learning Objectives—Students will learn...

- how to create and interpret dotplots.
- how to describe the center, shape, spread, and outliers for quantitative variables.

Lesson suggestions

- Don't teach dotplots. Just sneak them in.
- Start by letting students explore different distributions. Here's a set of google slides that my friend Jeff E. made and were adapted by my co-teacher Tiffini G. (Note: these slides have a PearDeck add-on. PearDeck is excellent, but not free.)
- Let students try to describe graphs using their own vocabulary first. Then step in towards the end of the lesson and use some of their words and help them refine their vocabulary.
- Make sure students are communicating in context, *constantly*. Not "the graph..." or "the distribution...", but "the heights...", etc....
- I use the mnemonic device CUSS for center, unusual features, shape, spread.

Textbook suggestions

- This is a no homework night for students. I prefer, with big ideas like these, to use two days to build the ideas and to give students a break on the first day!

Additional resources
AP Classroom Daily Videos:

- 1.5—Quantitative Variables with Graphs
- 1.6—Describing a Quantitative Variable

Khan Academy practice:

- Reading dotplots and frequency tables
- Describing distributions

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 5—Describing/Comparing Quantitative Data Using Stemplots—CED1.5

Learning Objectives—Students will learn...

- how to create and interpret stem and leaf plots.
- how to describe the center, shape, spread, and outliers for quantitative variables.

Lesson suggestions

- Make sure students are communicating in context, *constantly*. Not "the graph..." or "the distribution...", but "the heights...", etc...
- You might chose to use the survey you took the first week of class. Look to see if one of the variables would make a nicely readable stemplot.
- Because I want to introduce a back-to-back stemplot during this lesson, I dive right into comparing center and spread, not just describing.
- I have long enjoyed the back-to-back stemplot on Population Growth 2010 (SMW 5e Chapter 4 #11) and usually use that problem as a way to introduce these graphs.
- Did you know that stemplots were invented by John Tukey as a quick way to make a graph of a small data set? And did you further know that Tukey was the thesis advisor for Paul Velleman, the co-author of SMW? It is cool that we are connected to Tukey and that his contributions are so recent! Here's a link to his Wikipedia page.
- After reading and describing the Population Growth stemplot, I like to have students make one by hand.
- My goal is to both teach a new display and to reinforce ideas around center, shape, and spread.

Textbook suggestions

- Chapter 3 #40, 43, 46
- Chapter 4 #12

Additional resources
AP Classroom Daily Videos:

- 1.5—Quantitative Variables with Graphs
- 1.6—Describing a Quantitative Variable
- 1.7.1—Summary Statistics for Quantitative Variables

Khan Academy practice:

- Reading stem and leaf plots
- Describing distributions

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 6—Describing/Comparing Data Using Histograms—CED 1.5

Learning Objectives—Students will learn...

- how to create and interpret histograms.
- how to describe the center, shape, spread, and outliers for a histogram.

Lesson suggestions

- I usually make this day a two-day lesson and I teach Stat-Plot and Stat-Calc-1-Var Stats on their calculators. That said, keep reading, as I am *much* more interested in my student's abilities to read an already made histogram then I am in spending much time dealing their calculators.
- Your graphing calculator is not your best friend. Histograms on calculators are irritating and a pain to copy onto paper. Thus, too much time with the calculator is counterproductive. For this day, you could consider flipping your classroom for learning the calculator commands. They are in the textbook and there are plenty of YouTube tutorials.
- Likewise, too many problems "by-hand" is not a good use your time. Try to find the Goldilocks middle point. Students make enough to develop understanding, but spend more time reading and interpreting already finished results.
- Make sure to take a pre-made histogram, point to one bin and then ask: "This bin is between 40 and 50 and is 13 high. What does that mean, in context?" If students can explain that, then they know what they're looking at.
- I like to use the Population Growth 2010 stem and leaf plot and have students enter both data sets into their calculator. Then we can make a histogram of each region, adjust the x-scale so the histogram and stemplot looks the same, reinforce how to read the stemplot, etc....
- Since we're looking at summary statistics, start discussing median as a measure of center (which you probably hinted at already) and range as a measure of spread.

Textbook suggestions

- Chapter 3 #7-11
- Chapter 4 #7, 8

Additional resources
AP Classroom Daily Videos:

- 1.5—Quantitative Variables with Graphs
- 1.6—Describing a Quantitative Variable

Khan Academy practice:

- Reading histograms
- Quiz on dotplots, histograms and stem and leaf plots
- Describing distributions

Video Clip

- This short skit from Sesame Street is fun.

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 7—Describing/Comparing Quantitative Data Using Boxplots—CED 1.6, 1.8 & 1.9

Learning Objectives—Students will learn...

- how to create and interpret boxplots.
- how to describe the center, shape, spread, and outliers for a boxplot.
- how to use the outlier rule.

Lesson suggestions

- A fun introduction to boxplots to have 15 students stand in the front of the room and ask them to guess a number about you (I have them guess how many total songs I have bought in my lifetime, CD's, iTunes, etc....). Ideally a variable where a few students will guess very large numbers. Arrange them from small to big. Name the 5 # summary (John, you're the median, etc....). Draw the boxplot. Test for outliers, describe, etc....
- Make sure students are communicating in context, *constantly*. Not "the graph..." or "the distribution...", but "the heights...", etc...
- Don't forget that boxplots are summaries—for big data sets! Using five numbers to summarize 12 numbers... That's just silly. Provide students with five number summaries of BIG data sets. That's the point. This might be a good time to use StatCrunch or some other software to look at a really big data set and then to compare multiple groups within it.
- Make sure that students understand that stats problems come in two flavors:
    - A list of data points
    - A set of summary statistics and/or a graph

  Entering a 5-number summary into the calculator as if these five numbers are data points is a fatal error!
- Here is a google slide deck for this topic (thanks Tiffini G!).

Textbook suggestions

- Chapter 3 #13, 16
- Chapter 4 #15-27 odd

Additional resources
AP Classroom Daily Videos:

- 1.5—Quantitative Variables with Graphs
- 1.6—Describing a Quantitative Variable
- 1.7.1—Summary Statistics for Quantitative Variables
- 1.8—Creating a Boxplot
- 1.9—Comparing distributions

Khan Academy practice:

- Describing distributions
- Comparing distributions
- Identifying outliers
- Boxplots and outliers

AP Free Response Question

- 2010 Form B #1 is a great question for this unit. Use as a formative assessment, a quiz, or perhaps on your Unit Test.

Song: *The Boxer* by Simon and Garfunkle is a fun choice.

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 8—Mean and standard deviation—CED 1.7

Learning Objectives—Students will learn...

- how to summarize quantitative variables using summary statistics.
- how to use the mean and standard deviation as measures of center and spread.
- whether or not a statistic is resistant to skewness and outliers and how this affects our choice of summary statistics.

Lesson suggestions

- By this time, students have actually been introduced to all sorts of summary statistics. You've introduced the 5-number summary and you've been discussing mean and median as useful measures of center. But now it is time to unveil the standard deviation.
- While I do show students the formula (which is written inexplicably in two different ways on the official formula sheet), more than anything else I emphasize this interpretation of the standard deviation: "the typical distance of the data from the mean". Bob Hayden shared this phrase with the CB discussion forum and notes that a variation of this phrase should be used throughout the course, as in, "the typical distance of the blank from the blank." I will expand on this idea in later posts. An interested reader might take the formula sheet in hand, find any one of the various standard deviations, and see if you can fill in the blanks!
- I don't find calculating a standard deviation by hand to be helpful. What I have found to solidify student understanding of standard deviation is this card sort. I also find a conversation about $n - 1$ to be unhelpful. If you want to hear how I plan to discuss degrees of freedom, you'll need to wait until the end of this blog.
- Here is a google slide deck with a PearDeck add-on (written by Tiffini G).

Textbook suggestions

- Chapter 3 #17, 29
- Chapter 4 #25-28

Additional resources
AP Classroom Daily Videos:

- 1.7.1—Summary Statistics for Quantitative Variables
- 1.9—Comparing Quantitative Distributions

Khan Academy practice:

- Mean vs Median
- Visually assessing standard deviation
- Comparing Distributions

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 9—Finishing up quantitative descriptions—CED 1.5 & 1.7

Learning Objectives—Students will learn...

- how moving one data point can affect summary statistics.
- how to read and interpret a cumulative frequency plot.
- how to use and interpret percentiles.

Lesson suggestions

- Use big data sets. Load up a big dataset in StatCrunch. Cumulative frequency plots and percentiles are best discussed with large populations. StatCrunch has pre-made datasets, many of which are quite large. For example, here is a dataset with over 700 international soccer players, including heights and weights.
- Begin Rant: Arguing over whether a percentile is less than or equal to (the CED definition) or strictly less than (in some sources) is silly. Percentiles are a way to divide up a large population into groups and to be able to describe one individual's place within that population. Thus, it is very meaningful to say "My dad's height is at the 32nd percentile compared to other men" or "My SAT score is at the 73rd percentile". But debating the location of a percentile for 13 data points is just silly. /EndRant 😜
- I typically spend only 25 minutes on cumulative frequency graphs, put one multiple-choice question with this graph on their test, and then don't worry about these graphs again until April. Students enjoy taking a worksheet, using a ruler, and marking off the medians, the quartiles, etc... Bonus tip: since these graphs are also called ogives, I always play *Jump, Jive, and Wail*, a song that makes everyone smile!
- Here is a google slide deck for this lesson (thanks, Tiffini G!).
- Before you tackle transformations of data in the next chapter, it good to have students to think about the effect of moving one data point. The old free response question, Robin's Tips, 2016 #1, is a great practice or test problem for this topic.
- Here's an activity to help wrap up with unit. Divide students into groups. Have each group make a different graph of a variable (such as *total number of photos on your phone*). Each group makes a different quantitative graph. Then the pros and cons of the different graphing choices are debated and discussed (Thanks to Robin L. for this idea!).
- If you're feeling like this unit is long, you are *right*! While the CED recommends 14-16 days for this entire unit (which is the first five chapters of SMW), I need many more days than that to finish these topics. And that's OK. I'll make some suggestions later for saving time. Don't worry about establishing a firm foundation for exploring data and strong communication. I give a quiz on categorical data, a test on both kinds of data, and then yet another test after I've covered the normal distribution. Your mileage may vary!

Textbook suggestions

- Chapter 3 #31-34
- Chapter 4 #29-32

Additional resources

Khan Academy practice

- The effects of moving one point
- Percentiles

Project

- Here is a link to my quantitative project. I use StatCrunch. You could use free online software that is copied and pasted into a document.

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 10—Transformations and z-scores—CED 1.10

Learning Objectives—Students will learn...

- how linear transformations of a data set affect summary statistics.
- how to use z-scores as a measure of position and for making comparisons.

Lesson suggestions

- Try this quick calculator experiment.
  - Write 7 fake quiz scores on the board, out of 20 points possible, and have students enter the 7 numbers in their calculators.
  - Students find the summary statistics for these 7 numbers and you put a few of them next to the data.
  - You then tell students you liked the quiz so much, you decide to make it a 100 point test score. Students use List Commands and make a new list that is 5 times the first list.
  - Ask students, "Do you think all of these summary statistics are now 5 times larger?" Students verify using 1-var stats.
  - Now suggest that every student gets 3 points extra credit. A third list is made by adding 3 to the second list.
  - Ask students, "Do you think the statistics are now 3 larger?". Using 1-var statistics, students see that measures of spread did not increase.
- I have made two videos illustrating these transformations. If you open my publicly shard cloud drive at mrmathman.com/stuff and open the Unit 1 folder, you will find these videos.
- I use only about half the class period for transformations and then transition to z-scores for the rest of the period. I try to say repeatedly "standard deviations above/below the mean".

Textbook suggestions

- Chapter 5 #5,6, 11, 12

Additional resources
AP Classroom Daily Videos

- Lesson 1.10.1—z-scores and percentiles

Khan Academy practice

- Calculating z-scores
- Comparing z-scores
- Transforming data

AP Free Response Question

- 2011 #1 is often described as an all-time favorite FRQ. I highly recommend you use this problem after you cover z-scores.

Desmos Activities

- Percentiles and z-scores (In our text, percentiles are in chapter 3. But they can fit well in this lesson. Or with cumulative frequency plots.)

- Transformations

Song Suggestion

- *The Cha-cha Slide* is a very fun choice. What happens when we add a constant to our data? Slide to the right... 😜

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 11—The normal distribution—CED 1.10

Learning Objectives—Students will learn...

- the characteristics of the normal curve.
- how to use the empirical rule to estimate normal areas.

Lesson suggestions

- Did you know that the inventor of the normal curve was Carl Gauss? And that his picture was on the 10 deutsche mark, along with a normal curve? Grab a picture with a quick web search and show your students!
- Having students draw, label, and calculate some empirical rule problems is very standard fare. I like to start with something like male height, because it is so familiar to students. The Cattle problems in SMW also make for a nice introduction (Chapter 5 #17 and following).
- I don't always find doing lots of calculations with the empirical rule to be very helpful. Finding the size of the tails makes sense to me (e.g., 16% and 2.5%). But beyond that feels counterproductive. Once they start using their calculators, they're not going to bother with the empirical rule for, say, areas between 1 and 2 standard deviations.
- You might choose to spend only a portion of your class period on this lesson and start lesson 12 on this day.

Textbook suggestions

- Chapter 5 #18-22 even, #25, 26

Additional resources

Khan Academy practice

- Empirical Rule

Song

- *ABC-123*, by Jackson 5. What better way to help students to remember to put 3 marks on each side of the mean?

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 12—The normal distribution, continued—CED 1.10

Learning Objectives—Students will learn...

- how find the percentage between two values for a normal distribution.
- how to find a value matching a given percentage for a normal distribution.

Lesson suggestions

- I'll just start with this: I think that teaching Table A is counterproductive and does not improve student learning on this topic. Students don't need to see a 1950's method of solving a problem. And our calculators already make this harder than it should be. Try a normal calculator on StatCrunch or any other piece of software. This isn't tricky. But Table A can make it feel like it is.
- I prefer to teach students how to use their calculator and to clearly label their work so that they get full credit on the AP test.
- If you read through the rubrics carefully, a well labeled sketch is enough to get full credit. In my opinion, the most helpful rubric for this topic is 2017 #3.
- To be clear, I have my students draw, label, and shade a sketch *and* write a full answer in context.
- This lesson takes a few days. At least one day to conquer finding the areas and another to work backwards.
- In my cloud drive at mrmathman.com/stuff, Unit 1, is a worksheet called z-scores. This is a useful set of practice problems, some of which require students to find a z-score first, and then do some algebra to find the final answer.

Textbook suggestions

- Chapter 5 #39-50

Additional resources

AP Classroom Daily Videos

- 10.1.2—The empirical rule and normal calculations
- 10.1.3—More normal calculations

Khan Academy practice

- Normal distribution, area above and below a point
- Normal distribution, area between two points
- Normal calculations in reverse

Desmos Activity

- Practice with cumulative frequency plots and normal problems

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 13—Describing scatterplots—CED 2.4

Learning Objectives—Students will learn...

- how to make a scatterplot with labels and scaled axes.
- how to describe a scatterplot by discussing strength, outliers, form, and association.

Lesson suggestions

- I really like using my Correlation Stations to introduce this topic. Having students rotate around the room and actively create scatterplots makes for a nice break in the monotony.
- If you are looking for a distance learning version of this activity, my friend Drew wrote a survey that we assigned for homework.
- I introduce the terms explanatory and response variable and emphasize the importance of labeling and scaling your axes.
- I use the mnemonic SOFA.
    - Strength: strong, moderate weak
    - Outliers: just tell me where they are (we'll discuss them more later in the unit)
    - Form: linear or curved (I know you have learned about all sorts of curves (exponential, quadratic, etc...) but I don't care!)
    - Association: positive or negative
    - Don't forget context!
- If students create data, save it to analyze later in the chapter.
- Here is a google form you could assign to collect data from your students.

Textbook suggestions

- Chapter 6 #1-8

Additional resources

AP Classroom Daily Videos

- 2.4.1—Explanatory/response variables and creating scatterplots
- 2.4.2—Describing scatterplots

Khan Academy practice

- Describing scatterplots
- Note: there are 3 more pre-AP sets of exercises, if students need to develop this skill more slowly.

Desmos

- This activity covers lessons 13 and 14

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 14—The correlation coefficient—CED 2.5

Learning Objectives—Students will learn...

- the properties of the correlation coefficient.
- how to interpret the correlation coefficient in context.

Lesson suggestions

- I call this Pirate Day!! I play music from the movies and Disneyland and even wear a pirate hat, earring, eye-patch, and hook. I tell bad pirate jokes. After explaining about the correlation for a while, I reveal the abbreviation *r*, (Arrrrr!), and the joke is complete.
- I really appreciate the explanation in SMW about using a father's height to predict a son's height. I walk through this explanation with my students, while showing them the formula on the AP formula sheet. We look at how first quadrant and third quadrant points add to a positive correlation, etc.... You can find this on page XXXX.
- I find calculating *r* by hand to be counterproductive.
- I discuss the correlation properties (SMW5e, page XXXX). Students need to know that switching the variables does not matter, there are no units, *r* is not resistant, etc....
- But the key to the lesson is developing their intuition. My favorite website to use is this site, but there are lots to choose from. We do a few together as a class and then they finish with a partner.
- At some point in this unit you need to discuss the difference between an association and causation. This can be a fine day to get that done (and it corresponds with one of the AP Classroom videos below), but frankly, you can do it any time during this unit. In fact, you might revisit it at different points during the year. This site has become rather popular for launching a discussion of this idea.

Textbook suggestions

- Chapter 6 #15-20, 31, 32

Additional resources

AP Classroom Daily Videos

- 2.5.1—Using correlation
- 2.5.2—Correlation or causation?

Khan Academy practice

- Correlation coefficient intuition

Desmos

- This activity covers lessons 13 and 14

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 15—Least squares line, predictions, and residuals—CED 2.6

Learning Objectives—Students will learn...

- to use a regression line to make a prediction.
- to find and interpret a residual in context.

Lesson suggestions

- Let's talk. Regression on calculators is *terrible* (StatsMedic agrees). The AP test has *never* asked students to press the LinReg button their calculator. Ever. I haven't taught my students to do regression on their calculator for over five years. And frankly, I have a hard time of thinking of a teaching decision that has made a unit better than this change. Here's what I do.
- I start with a really big, very interesting dataset. You can use this PearDeck to guide your lesson (note: these slides cover lessons 15 and 16).
- The above PearDeck uses this amazing dataset in StatCrunch. These data are 5,222 movies with the movie title, budget, domestic gross, and international gross. We start by describing the scatterplot using budget to predict domestic gross.
- We use StatCrunch to find the regression equation and use it to make a few predictions.
- The next step is the best. I ask the class to pick a movie that over-performed expectations. They pick a title, we make a prediction, and then they find a residual—without me ever giving them a formula or instructions about what a residual is! They just say something like "Movie XX made $43 million more than expected." We do the same thing with a movie that did terrible. *Then* the formula for a residual is introduced and formalized.
- Play some movie soundtrack themes during this lesson. From *Star Wars* to *Harry Potter*, there are so many great choices that make students smile while they are investigating the highs and lows of different movies.

Textbook suggestions

- Chapter 7 #1-4, 31bc, 32bc

Additional resources
AP Classroom Daily Videos

- 2.6.1—Making predictions
- 2.7.1—Residuals

Khan Academy practice

- Calculating and interpreting residuals

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 16—Interpreting slope and y-intercept—CED 2.8

Learning Objectives—Students will learn...

- how to interpret the slope and y-intercept in context.

Lesson suggestions

- Use the class example for residuals and now build further understanding with slope and y-intercept.
- It is crucial that students use predictive language in these interpretations. Ideally, they will use terms like "predicted" or "expected". Terms like "approximately" or "about" are acceptable, but in my opinion do not communicate as strongly (and were not accepted for full credit for 2015 #6).
- Slope: For every one more x, we predict slope more y's.
- y-intercept: When x = 0, we predict y.
- I provide these two templates and then encourage deeper understanding with more examples.
- You can take equations from your opening lessons in this unit and add these interpretations.

Textbook suggestions

- Chapter 7 #5, 6, 13, 14, 25, 26

Additional resources

AP Classroom Daily Videos

- 2.8.2—Interpreting slope and y-intercept

Khan Academy practice

- Interpreting slope and y-intercept

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 17—Interpreting R-squared—CED 2.8

Learning Objectives—Students will learn...

- how to interpret R-squared in context.

Lesson suggestions

- The *Teacher's Resource Guide* that comes with SMW has the best explanation of R-squared that you will find anywhere. Every time I share it with teachers, they are always so impressed. It uses pizza pricing to explain this concept—find it and use it!
- I don't find the mathematical side of R-squared to be helpful for students. The AP test does not explore these ideas and these concepts are really better served in a higher level stats course. Thus, I choose to not spend time on what R-squared is a percentage of, etc. ... Your mileage may vary!
- Spiral, spiral, spiral! Pick a new regression setting. Interpret slope, correlation, find a residual, all in context, or course. And then add R-squared. Or spend some time on this lesson and then also add the next lesson. Remember that regression is only a small portion of the overall exam (only 5% to 7% of the multiple-choice!), so resist the temptation to over-explain these concepts.

Textbook suggestions

- Chapter 7 #7, 8, 28, 30

Additional resources

AP Classroom Daily Videos

- 2.8.3—R-squared

Khan Academy practice

- R-Squared Intuition

Desmos

- R-squared (and residual plots)

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 18—Interpreting a residual plot—CED 2.7

Learning Objectives—Students will learn...

- how to interpret a residual plot to confirm a linear model.
- to identify problematic residual plots.

Lesson suggestions

- Save time by providing students with pre-made residual plots. Don't take time to make them with technology.
- Use the definition of the word residual—leftover. The residual plot is showing the leftover variation. Hopefully, all that is left is randomness. If a pattern is leftover, we missed something!
- Show students residual plots with increasing or decreasing variation and discuss why this is also problematic.
- This might be a great day to finish with an old free-response question.

Textbook suggestions

- Chapter 7 #11, 12, 37, 38

Additional resources

AP Classroom Daily Videos

- 2.7.1—Interpreting residual plots
- 2.72.—More on residual plots

Khan Academy practice

- Residual plots

Desmos

- Residual plots (and R-squared)

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 19—Computer output and math fun facts—CED 2.8

Learning Objectives—Students will learn...

- how to read computer output for linear regression.
- the meaning of the term "least squares".
- why the least squares line passes through the mean-mean point.

Lesson suggestions

- There are lots of ways to make sure that students understand the mathematics beneath the regression. I prefer to sprinkle them throughout the unit.
- Students should know that the regression line minimizes the sum of the residuals squared.
- A simple example or two will demonstrate the logic of the mean-mean point. e.g., "A student who studies an average amount should have an average GPA." or "A person of average height is predicted to have an average weight."
- After a few days with our Chromebooks and software, I pivot to computer output. I use a combination of textbook exercises, old free-response questions, and problems that I create. You can find a regression equation on your calculator and then modify the output to match the AP exam. See the list of FRQ's below for examples of the format.
- Here is a regression project. Enjoy!

AP Free Response Problems

- Waiting in line—2018 #1
- Gray wolves—2017 #1
- Semesters and salary—2016 #6
- Residuals and FCR—2014 #6
- Pro-tip! It is easy to modify a regression question. Take any of the above problems and just add some extra parts. Interpret the slope, find and interpret a residual, etc... This can be great for test security, also.

Textbook suggestions

- Chapter 7 #37-42, 47, 48

Additional resources

AP Classroom Daily Videos

- 2.8.1—Least Squares
- 2.8.3—Computer output

Khan Academy practice

- Using least-squares regression output

Desmos

- Regression practice

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 20—Outliers, influential points, and high-leverage points—CED 2.9

Learning Objectives—Students will learn...

- the definitions of outliers, influential points, and high-leverage points.
- how to identify when a point has influence and/or is a high-leverage point.

Lesson suggestions

- Read the new definitions the CED carefully. The definition of an outlier has been narrowed (large residual) and the definition of an influential point has been broadened (can change the "relationship" substantially).
- The wandering point activity in the *Teacher's Resource Guide* has been adapted for Desmos. I find that about 20-30 minutes of exploring this activity, followed by a few notes is plenty of time for this lesson.
- Using software to explore how one data point can be influential is the best way to approach this lesson. There are numerous applets where students can explore this effect.
- 2003 Form B #1 is a great problem for teaching or assessing these ideas.

Textbook suggestions

- Chapter 8 #17-20, 31

Additional resources

AP Classroom Daily Videos

- 2.6.2—Dangers of extrapolation
- 2.9.1—Outliers and high-leverage points

Khan Academy practice

- Effects of influential points
- Identify influential points

Desmos

- Wandering point activity

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 21—Departures from linearity—CED 2.9

Learning Objectives—Students will learn...

- how to read and interpret a non-linear regression that has been transformed to achieve linearity.

Lesson suggestions

- This topic has been on the AP exam very, very infrequently. The secure exam release has revealed zero multiple choice questions on this topic. Of course, we don't get to see all the tests. But given the scant attention that the AP exam has given to this topic, most teachers have dramatically reduced the amount of time they spend on this topic.
- Some years, I have taken about 30 minutes in May, right before the exam, to give students a quick crash course on this topic.
- More recently I have decided to spend one day at the end of this unit to introduce this topic. I posted this lesson on my blog. Here is the link, which includes a handout for students.
- The 1997 Investigative Task, #6, is about this topic. I enjoy using this problem—but more as a teaching example (students try the problem, then I explain some things about it, no grade, etc....).

Textbook suggestions

- Chapter 9 #11, 13, 19

Additional resources

AP Classroom Daily Videos

- 2.9.2—Transformations for linearity

Khan Academy practice

- Transformations to achieve linearity

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 22—The Importance of Random Sampling—CED 3.3

Learning Objectives—Students will learn...

- the importance of random sampling to obtain a representative sample.
- that bias occurs when convenience samples are collected.

Lesson suggestions

- Many years ago Doug Tyson wrote an excellent activity for CollegeBoard entitled "Show Me the Money". You find these materials on his website.
- For this activity, you begin by asking students about their favorite movie, what they think were the top grossing movies in the previous calendar year, and then their estimate of the average gross of the top 200 movies of the prior year.
- Students are given a list of the top 200 movies from the prior calendar year (This list is updated annually by all sorts of folks. I usually keep an recent file in my cloud drive.)
- Students first list 10 movies that they saw or wished to see. They find the average of those 10 movies. Students place their average on a class graph.
- Next, students collect a random sample and these averages are placed on the board.
- The key moment of the lesson is contrasting the biased sample vs. the random sample.
- For my students this is often the first time they use a random digit generator. I like using random.org. It is a more robust tool and is easy to use (there is, in my estimation, virtually no chance that a student would need to access the random commands on their calculator—it simply wouldn't be verifiable on a test). I do like to discuss with my students how true randomization is difficult to attain, computers only make pseudo-random numbers, etc... I also really enjoy showing them the $60 book on Amazon with a million random digits. Researchers value true randomness!
- Finally, if you want to add two bits of fun to the lesson, here are two suggestions. For one, play your favorite movie soundtrack while students collect their samples. For two, the review on Amazon for the random digit book are hilarious. Worth reading a few for the class.

Additional resources

AP Classroom Daily Videos

- 3.1—Collecting unbiased data

Khan Academy practice

- Identifying population and sample

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 23—Survey Designs—CED 3.3

Learning Objectives—Students will learn...

- the advantages and disadvantages of various sampling methods.
- how collect simple random samples, stratified random samples, cluster samples, systematic random samples, and a census.

Lesson suggestions

- I would encourage you to be careful how much time you spend on this unit. The vocabulary is dense, the concepts can be complex, and students can find the ideas difficult. However, it is worth noting that this unit is the smallest portion of the AP Exam.
- Thus, to keep the pace brisk, I have worked hard to teach all the various survey methods in one class period. I usually start with a scenario and have students brainstorm how they would collect a survey. Their ideas can be used to introduce some of the methods and then I supplement with the remaining ideas.
- I like to start with an on-campus survey. Students have a lot of prior knowledge that is helpful in building the ideas. Classrooms are natural clusters, grade levels are a reasonable choice for strata, etc....
- This is also a great unit to encourage students to read the textbook. Students need repeated emersion with this vocabulary. Flip your classroom and have them read first. Or have them watch YouTube videos introducing the concepts. Or Khan Academy.

Textbook suggestions

- Chapter 11 #5, 7-10-13

Additional resources

AP Classroom Daily Videos

- 3.3.1—Random sampling methods
- 3.3.2—Pros and Cons of sampling methods

Khan Academy practice

- Simple random samples
- Sampling methods

Desmos

- Sampling Methods

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 24—Biases in Surveys—CED 3.4

Learning Objectives—Students will learn...

- how bias can occur in sampling.
- how voluntary response, undercoverage, and nonresponse can create bias in surveys.
- the various ways response bias can occur.

Lesson suggestions

- Again, I like to start class with an accessible example and have students discuss various forms of bias that they see. Then I can build in the correct vocabulary to accompany their intuition.
- For a class example on this day, I like to introduce something bigger and broader. For example, a recent election or civic issue that your county or state has considered makes a strong example. This encourages students to think more carefully about the population of interest (adults/voters in the given region) and helps them broaden their thinking beyond the parts of the world that are most interesting to them.
- Make sure that students are being specific. Students tend towards vague statements (e.g., "Some people will lie..." or "Some students won't answer the survey..."). I have found it valuable to collect weak student responses (often on a quiz) and then contrast weak replies with full credit answers. If you navigate to the Unit 3 folder of my cloud drive, you will find a document entitled "What You Said" that I have used with students. This is an example. Your students will learn best if you use their answers.
- Nonresponse bias is subtle. Students need to understand that this bias only occurs if the nonrespondents are systematically different than the respondents in some way.

Textbook suggestions

- Chapter 11 #17, 23-26, 29, 30

AP Free Response Questions for Surveys

- 2011 #3—Carpets, clusters vs. strata
- 2013 #2—University appearance, strata (Warning: part (c) is challenging! Focus on discussion and learning. Most students missed this on exam, which is OK. It just means that sometimes students are asked challenging questions, which will help the 5's earn a 5 and not really affect the scores of the rest of the students.)
- 2004 Form B #2—Dining hall, selection and response bias. This is an older problem. Old problems are fine to use, I just focus more on the content and less on the details of the rubric.

Additional resources

AP Classroom Daily Videos

- 3.4—Problems with sampling

Khan Academy practice

- Bias in samples and surveys

Desmos

- Biases in Surveys

Project

- There is probably no more popular project in all of AP Stats than the Bias Project. First proposed by Dick Schaeffer, the idea is simple but extremely effective. Students conduct a survey and ask an unbiased question. Then they repeat the survey (using the same random sampling method), but create response bias. They add extra information to their question, they change their appearance, etc. … My cloud drive has instructions, a rubric, and examples. I have students make either a powerpoint, a poster, or a movie. I think students should collect a survey in person. They are invariably surprised at the non-response rate. They are also often surprised at how well response bias can succeed. Finally, I have students present to each other in small groups. I find presentations to the whole class to both take too long and, frankly, to be very boring for everyone involved.

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 25—Observational Studies—CED 3.2

Learning Objectives—Students will learn...

- the difference between a prospective and retrospective observational study.
- the difficulty of confounding variables when drawing conclusions from observational studies.
- the power of observational studies for finding associations that may lead to important conclusions.

Lesson suggestions

- This is a great topic to introduce with ... quiet! Give students an article from Pew Research and have read, silently!, for the first 10 minutes of class. (Here is an example. And another.)
- Explaining confounding can be challenging. It is critical that students understand that a confounding variable is both linked to the explanatory variable and the response variable.
- Here's one example: people who join a running club report an increase in happiness. If we consider the exercise to be the explanatory variable, then the increase in friendships/community would be a confounding variable.
- The term "lurking variables" frustrates some teachers (especially because of the way it was handled in one older rubric). I never use the term. It does not have any universal statistical definition and I don't find it helpful. I do discuss "extra" variables. Or, if you'd like to be a bit more formal, extraneous variables. When you discuss confounding, students will quickly provide you with variables that are extraneous. For example, in the running club example, students might say, "Maybe the runner is happier because they got a job!". Well ... maybe. And employment is connected to happiness. But that's just an extra variable running around that aren't measuring (and it's not connected to the explanatory variable, so it is not confounding!). There are lots of extra variables running around making a mess of observational studies.
- The messiness of extra variables should lead us to two important ideas. Number one, experiments are great precisely because they (mostly) control these pesky extra variables. Number two, don't be too scared by them! Observational studies are cheap and can have very large, broad samples, leading to important and useful observations about how things are connected. Once we observe those connections, we need to be careful about our conclusions. But these connections point researchers in important directions!

Textbook suggestions

- Chapter 12 #1-4

Additional resources

AP Classroom Daily Videos

- 3.5.1—The problem of confounding

Desmos

- Observational Studies and Experiments

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 26—Designing Experiments—CED 3.5

Learning Objectives—Students will learn...

- the principles and vocabulary for experiments.

Lesson suggestions

- Because I come from many generations of farmers, I like to ask my students to design an experiment where a farmer tests a new, more expensive food for his pigs. Students have plenty of ideas. It usually takes most of the period to outline a completely randomized design and to introduce the terms explanatory variable/factor, treatment, level, response variable, etc. ...
- Students can become confused because treatments and levels sound the same. But that is because they are! Read the CED carefully. VAR 3.A.2—"The levels ... are called treatments." If you're formulating a quiz question about this term and your experiment has only one factor, don't ask two questions, just ask one: "What are the levels/treatments." This will help your students avoid confusion. And then try to wait a few days before introducing experiments with a combination of levels of different factors.
- The Memory Game (see cloud drive, unit 3 folder) can be used to introduce experiments. Students are given instructions about memorizing a list of statements. You read 25 statements. Then students are given a 20 question quiz and they see how many details they remember. What they don't realize is that half the class was given instructions that advised them to visualize the statements they hear, while the other half did not. You can collect the class data, compare the two groups, discuss random variation vs. statistical significance, etc. ...

Textbook suggestions

- Chapter 12 #5, 6, 21, 29

Additional resources

AP Classroom Daily Videos

- 3.5.1—The problem of confounding
- 3.5.2—Designing an experiment

Desmos

- Observational Studies and Experiments

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 27—More on Experimental Designs—CED 3.5 and 3.6

Learning Objectives—Students will learn...

- the advantages of using block and matched pairs design for experiments.

Lesson suggestions

- Explaining the concepts of blocking and matched pairs are very challenging. I usually opt to spend one day on blocking and the next on matched pairs.
- Students tend to want to compare the groups created by the blocks. And I have been told by people smarter than I that statisticians do this. But AP rubrics have focused on using blocks to reduce variability in the response variable and create treatment groups that are as similar as possible.
- There is a long-standing blocking activity called Rolling Down the River. You can find it here. I have created a google sheets version of this activity. I also made the data more realistic when I created it. You can find that activity at bit.ly/rollingriver.
- Guide students to write careful conclusions. More on this concept in Lesson 29. But as you'll hear me say in #29, conclusion writing needs to be practiced throughout the unit.
- To introduce matched pairs, I'll suggest the simplest experiment one can devise. Ask students if they snap faster with their dominant hand. Have students raise their dominant hand. Then instruct students whose birthday is an even day of the month to switch hands, thereby randomizing the order of the treatments. Students snap for 10 seconds and record their total. Then the other hand. Record the data. Save for a matched pairs t-test. Discuss paired differences.

Textbook suggestions

- Chapter 12 #19, 20, 30-33

Additional resources

AP Classroom Daily Videos

- 3.5.3—Experimental designs
- 3.6.1—Blocking
- 3.6.2—Matched pairs

Khan Academy practice

- Experimental designs
- Experimental design considerations

Desmos

- Observational Studies and Experiments

Exit ticket: This link (blocking) or this link (matching), will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 28—In Class Experiments—CED 3.5

Learning Objectives—Students will learn...

- how to fully implement an experiment.

Lesson suggestions

- There are many experiments you can run in your class. I have listed some of my favorites in earlier posts. Here are a few more.
- Matched pairs emoji tracing. Give students a paper with the same emoji printed twice and very large. I saw someone use the poop emoji and that's what I do. If you would like to be more serious, have at it. Supply students with a marker or have them grab a colored pen. Students time how long it takes to trace the emoji. They can pair up to run the stopwatch. The comparison is dominant hand vs. non-dominant hand. Students should randomize the order of the treatments, of course.
- If you navigate to my cloud drive, you will find the melting chips experiment. Students are given three different kinds of chocolate chips (milk, dark, peanut butter, white, etc...). Students measure the time it takes for each chip to melt. Your classroom becomes delightfully quiet while the melting occurs (students have the chip pressed with their tongue against the roof of their mouth). I seize the opportunity to play the song "*I'll stop the world and melt with you*".
- Some teachers compare pulse rate change after drinking soda. Half the soda has caffeine and half does not.
- You can also measure pulse rates using a phone app. One option is to compare standing vs. sitting pulse rate.
- We had an outdoor earthquake drill one day, so I had my students do all sorts of outdoor trials. For each trial they had to describe the explanatory variable, the treatments, and the response variable. I'm afraid I didn't keep any notes on this day. It was a fun class period.
- Speaking of fun, class experiments are fun! But they can also take time and result in only small learning gains. I recommend you pick a few anchor activities and resist the temptation to spend too much class time on activities.

AP Free Response Questions for Surveys

- 2019 #2—Fungus—Careful explanation of core vocabulary, implementing random assignment, control groups
- 2016 #3—Smoking and Alzheimer's—Observational studies, confounding, explanatory/response variables
- 2009 #3—Frog dissection—the importance of random assignment

Desmos

- Conclusions and Experiments

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 29—Making the Right Conclusion—CED 3.2 and 3.7

Learning Objectives—Students will learn...

- the nature and scope of the conclusions from different types of studies.

Lesson suggestions

- This lesson could be its own lesson, or it can enfolded into various lessons as the topic naturally arises.
- It is crucial that students understand that causal conclusions require an experiment.
- It is also important for them to understand that you can only generalize to the population from which you randomly sampled. Unfortunately with experiments, this means an initial conclusion is restricted to people who are "like the volunteers".
- I have grown to be more enthusiastic about observational studies than I used to be. Yes, it is true that we can only make conclusions about associations. However, observational studies are cheap and can be collected out of very large populations. So these associations can give researchers promising starting points to examine. And that research often leads to experiments and strong causal claims.

Textbook suggestions

- Chapter 12 #7-16, 35, 36

Additional resources

AP Classroom Daily Videos

- 3.2—Types of studies and conclusions
- 3.7—Conclusions from experiments

Khan Academy practice

- Generalizability of results
- Identifying different types of studies
- Conclusions from observational studies vs. experiments
- Finding errors in study conclusions

Desmos

- Conclusions (and more)
- Cumulative Activity

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 30—Probability & Randomness—CED 4.3

Learning Objectives—Students will learn…

- the nature of randomness.
- difference between the law of large numbers (valid) and the law of average (invalid).
- that in long run experimental probabilities will become closer and closer to the theoretical probabilities.

Lesson suggestions

- Students don't understand randomness. They don't realize how many streaks happen in the short run (demonstrating that the law of averages is untrue—that deck of cards is *not* "due"). They don't realize how long it can take the theoretical probability to drown out an initial streak in the short run.
- One class activity I have used is having students write down how they think a coin will act over the course of 20 flips. Then all students report their longest streak. Most students think that 3 is the longest. Now use a site like random.org and have students generate a streak of 20 1's and 0's. Report the longest streak. Streaks of 5 will not be unusual!
- You might introduce the basic fundamentals on this day, which students will certainly have encountered prior to this course. Probabilities need to sum to 1, be between 0 and 1 inclusive, and the complement rule. Since our text uses the term "legitimate probability distribution", MC Hammer's "Too Legit to Quit" makes an irresistible (if quickly irritating) song choice!
- You can also give students a sneak preview of expected value by adding insurance and/or casinos to the discussion about the law of large numbers. Some (too many!??!) students are oddly surprised to hear that the odds in a casino are set against the gamblers.
- "*Take a Chance on Me*" by ABBA is simply the greatest probability song of all time!

Textbook suggestions

- Chapter 13 #1-10

Additional resources

AP Classroom Daily Videos

- 4.1—Random vs. non-random
- 4.3—Intro to probability

Khan Academy

- Theoretical vs. Experimental Probabilities

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 30B—Thoughts about teaching probability and a dream about the future.

My friend Penny Smelzter and I were chatting about teaching probability. She gave me advice that drastically changed how I teach the rest of this unit. She observed that students find any single probability concept fairly simple, but they struggle when they have to choose between the various topics. Indeed, as I reflected back on my own experience learning probability I realized that this was true.

How did Penny adjust her teaching to fix this problem? She teaches as many concepts as she can fit into one class period and then uses the following class period to give students spiraled practice. Teach, spiraled practice, repeat. And the spiraling grows to include all possible topic as you progress through the unit.

Unfortunately, I don't have a wonderful set of materials to provide. I have made some very ugly worksheets where I have used brute cut-and-paste to combine all the probability chapters. Pending time and resources, I may create such spiraled worksheets as this blog continues. Or, perhaps, such a resource will be created to accompany SMW 6e.

I don't think there is any silver bullet for this unit. I should also note that this is the one topic that I wish the CED would change. I think it is too many topics given the time allotted. For students who have a strong honors-math background and/or an innate sense of how probability works, the task is doable. For students who come from a less robust background (the type of students that CB encourages us to be inclusive and to include in our programs), the task is much more daunting. Finally, I would note that the 2016 GAISE report for colleges and universities calls for a de-emphasis on probability and the inclusion of multivariable analyses. Likewise, the new 2020 K-12 report suggests that multivariable analyses be threaded throughout K-12 statistics education. If this educator could be allowed to dream for a minute, he envisions a CED with a reduction in theoretical random variables and the inclusion of multivariable linear regression. And lest you think I'm too crazy, see Intro Stats, 5e, as a text that follows these new guidelines and does precisely this.

Lesson 31—Basic Probability Concepts—CED 4.3-4.6

Learning Objectives—Students will learn…

- the principles of simple addition and multiplication rule.
- how to use the complement.
- the meaning of mutually exclusive and independent.

Lesson suggestions

- Basic probability ideas are taught at numerous grade levels prior to AP Stats. This might be a lesson to consider flipping. But at the least, plan on a lesson that is light on lecture and heavy on students doing the heavy lifting.
- Most students intuitively know to add for "or" problems and multiply for "and" problems. With a small amount of guidance they can dive into the Chapter 13 problems.
- "At least one" problems deserve some time and instruction. The rhyme "at least one is one minus none" will help.
- The terms mutually exclusive and independent are important. However, they will mean more when you start doing problems that *don't* have these traits. Thus I introduce them here and wait until the next few lessons to define these ideas more completely.
- As per the prior post, consider spiraling. One option would be to teach this entire chapter in one big lesson and then have a practice day immediately following.

Textbook suggestions

- Chapter 13 #11, 15, 21-26, 39, 44

Additional resources

AP Classroom Videos

- 4.5—Independent probabilities

Khan Academy practice

- Basic Addition Rule
- Independent Probability
- At least 1 probability

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 32—Probability Rules on 2-way Tables—CED 4.6

Learning Objectives—Students will learn…

- how to find probabilities from 2-way contingency tables.
- how to use the general addition formula.

Lesson suggestions

- Students found percents from 2-way tables in Chapter 2. Now it is time to make this process a bit more formal. While in Chapter 2 I avoid using the term "probability" explicitly, here is the time.
- Lest you think I completely despise probability in our course, this is the lesson that I think we need to lean into most heavily. Reading and interpreting probability from data tables is precisely what our course should be about.
- Again, lean into the students' intuition. They can read 2-way tables. One fun option is to create a table on the board and have students fill it in with tally marks. For instance, create a table with "Do you play a sport" and "Do you have a car"? After students supply the data, you can ask the typical questions.

  What is the probability that a randomly chosen student in this class…
  - plays a sport and has a car?
  - plays a sport or has a car?
  - has a car, given that you know they play a sport?
  - doesn't play a sport or have a car?
- This is the time to formalize the general addition rule. Point out to students that there are 3 different ways to calculate an "or" probability. Students should see that while the mutually exclusive overlap did not happen last chapter, now this is a problem. A Venn diagram might be added to the problem. Students should be pointed to the general addition formula on their formula sheet.
- Speaking of formula sheet, make sure your students have one! Using the formula sheet for the first two units isn't really of much use. But now it becomes crucial. I give all my students a copy of their own with a sheet protector. I instruct them to annotate on the sheet as we progress through the rest of the course. For all quizzes and tests I have a clean set of sheets that I distribute.
- Students will approach this probability rule differently. Some will find a formal definition and formula helpful. Others will find it confusing. Encourage students to use the approach that makes the most sense to them. But also look for opportunities to demonstrate both methods (formula vs. intuition). As you continue spiraling probability problems, students will grow in confidence in using an abstract formula.

Textbook suggestions

- Chapter 14 #15, 16, 19, 20

Additional resources

AP Classroom Daily Videos

- 4.4—Mutually exclusive
- 4.5—Conditional probability

- 4.6—Probability of a union of two events
- 4.6—Probability rules and FRQ's

Khan Academy practice

- Addition Rule
- Multiplication Rule
- Conditional Probability

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 33—Tree Diagrams—CED 4.5

Learning Objectives—Students will learn…

- how to construct a tree diagram to calculate probabilities.

Lesson suggestions

- This is a straightforward topic that deserves some class time. Students need guidance in deciding when to use this method, so time should be devoted to contrasting tree diagram problems (which dependent probabilities provided) vs. other kinds of problems.
- The most frequent use of tree diagrams on the AP Exam usually finish with the calculation of a conditional probability.
- Given that this lesson isn't too long, if you haven't given a spiraled set of problems yet, this is a good chance! If you're not familiar with Marzano's conclusions on effective instructional strategies, let me mention this briefly. When students sort and classify information they learn more than just drill and practice. A LOT more. Consider giving students a mixed-up set of problems and then have them sort them into the different strategies and types of problems. I will have more to say about this strategy for teaching inference. But let me emphasize a few points.
    - Spiraled practice is essential for learning probability.
    - Choosing the right procedure isn't a small thing in statistics. Not only is it one of the biggest challenges on the AP Exam, it is a real life challenge as well. Chances are your students will go off into the world and be given stats software and data to analyze. The hard part won't be reading the results or deciphering formulas. The most difficult and important decision will be choosing the right procedure to run for the data at hand.
    - Finally, my good friend Doug Tyson has said, "In calculus, it is easy to know what to do, the hard part is doing it. In statistics, the hard part is deciding what to do. After you pick the right procedure, doing it isn't too hard." #truth

Textbook suggestions

- Chapter 14 #23, 24, 51-56

Additional resources

AP Classroom Daily Videos

- 4.5—Conditional probabilities

Khan Academy practice

- Tree diagrams & Conditional probability

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 34—Dependent Probabilities—CED 4.5

Learning Objectives—Students will learn...

- how to calculate the probability of dependent events.

Lesson suggestions

- Jimmy Fallon plays a game with his guests called Egg Russian Roulette. My friend James Bush realized that this is a great opportunity for teaching dependent probabilities. Here is one sample with David Beckham. YouTube can provide you with many more.
- I have done with activity myself. Both against students and against our principal. If you're up for a little messiness, I recommend it.
- This is an opportunity to introduce the 10% condition. Jimmy is sampling more than 10% of the population. This leads to a different calculation than the M&M problems in chapter 13 where we could just multiply independent probabilities without a care! Students should be guided to contrasting these two different applications of multiplication.
- I'll say it again, spiral your practice. How best will students see the difference between independent and dependent probabilities? If they have some questions from both Chapter 13 and 14, ideally mixed up so they have to tell the difference.
- And, of course, these leads to the general multiplication rule, which is on the formula sheet. You may have done this on the prior lesson also, but you want to connect the intuitive understanding (4/12*3/11) with the general formula.
- Students also need to formalize checking independence on a two-way table. Students need to be able to evaluate a 2-way table for independence using the definition P(A) = P(A|B). I call this the "dumbbell test", because when you circle the events described by the rules and draw either the horizontal or vertical lines connecting them, it makes a pair of dumbbells. Students find this to be useful mnemonic device. Here is an example from 2019 exam, #3.

The "dumbbell test"

	Never	Sometimes	Always	Total
Men	0.0564	0.2016	0.2120	0.4700
Women	0.0636	0.1384	0.3280	0.5300
Total	0.1200	0.3400	0.5400	1.0000

Textbook suggestions

- Chapter 14 #19, 20, 25-30, 41, 42

Additional resources

Khan Academy practice

- Dependent Probabilities

- AP Free response problems
    - 2011 #2—politics & gender—checking for independence (I adore this problem. I like to use parts (a) and (b) as an individual quiz and see if they can solve part (c) in groups. Sometimes I use this problem in August with chapter 2. Some years I wait until this unit.)
    - 2017 #3—melons—normal calculations and tree diagrams.
    - 2017 #6—chips vs. coins—a *very* challenging Investigative Task.

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 35—Expected Value & Standard Deviation of Random Variables—CED 4.7 & 4.8

Learning Objectives—Students will learn...

- how to calculate the mean/expected value of a random variable.
- how to interpret the mean/expected value of a random variable.
- the importance of the law of large numbers for settings such as casinos and insurance companies.

Lesson suggestions

- I like to start this lesson with a simple game. For example:
    - Student government has a fundraiser at a Halloween carnival.
    - $5 ticket buys you the chance to draw 1 card.
    - Draw a face card, win a t-shirt ($7 value).
    - Draw an Ace, win $20.
    - Any other card, no prize for you!
- We find the expected value by hand. Discussion ensues about the long run profit. What if only 5 people play? 100 people? 1,000 people?
- The second problem is usually an example from the text. First by hand, then add the calculator and use 1-Var Stats to arrive at the same value.
- We discuss the Law of Large Numbers and how the expected value is the long-run average. We also discuss how this idea is essential for the insurance industry and Las Vegas.
- Time to introduce standard deviation. We use the formula sheet, write down two terms, and I point out what the formula is doing. We contrast this formula with the regular standard deviation formula.
- I don't find it is helpful to force students to calculate one by-hand. What I do find helpful is to say something like this:
- "This is the first of many formulas (on the formula sheet), where you will let the calculator do the work, but then you will show what formula the calculator was using. This demonstrates that you understand what your technology is doing."
- If you glance down your formula sheet, you can see this list: binomial pdf's, standard errors for inference, etc. ...
- Thankfully, students have an easier time of knowing when to use these formulas than some of the other probability topics. Random variable tables have a rather unique appearance. Also, as the unit progresses, I emphasize the word "expected" as the cue.
- This lesson, as written here, is a bit long. You may need three days to teach lessons 35 and 36.
- Finally, it is crucial that students are reminded that expected values are not values that will be observed, but are long-run averages, and thus should not be rounded. This error is always penalized on the AP test, as it reflects a fundamental misunderstanding of expected value. A nice example to help students is GPA. If you have a 3.8 GPA, did you earn a 3.8 in a class? No. You averaged 3.8 across all your classes.

Textbook suggestions

- Chapter 15 #3-6, 19, 20

Additional resources

AP Classroom Daily Videos

- 4.7-Intro to random variables
- 4.7-More on random variables
- 4.8-Expected value & sd

Khan Academy practice

- Expected Value
- Standard Deviation

Desmos

- Expected Value
- Standard Deviation

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 36—Building & Interpreting Random Variable Models—CED 4.8

Learning Objectives—Students will learn…

- how build a probability model for a random variable.

Lesson suggestions

- I have mixed emotions about spending class time on this topic. For students who have a strong math background, this lesson can help solidify their understanding. For students who have a weaker background, I feel this lesson can be a real struggle. And when I'm focused on inclusion and equity in my program, theoretical probability (like this) is the first thing I'm willing to skip. I also note that on the AP Exam, students have almost always been provided with the probability model when they need to calculate expected value.
- That said, I do like the textbook examples in *SMW* on this topic. You can practice tree diagrams, conditional probability, etc. … (see the list below). So, depending on the year and my mood, we usually explore a few of these problems.
- These problems work well with a Think-Pair-Share model. Have students try to build the model, discuss in small groups, then show the model for the entire class. I use 1-Var Stats to find the mean and standard deviation and make sure to reinforce the idea of the long run average, not rounded.
- The probability model ***must*** add to 1. This seems obvious. But it is not obvious to students. Any model that does not add to 1 on the AP exam (traditionally) cannot receive a Partial, but is marked Incorrect.

Textbook suggestions

- Chapter 15 #7-18, 21-28 (pick just a few!)

Additional resources

Khan Academy practice

- Random Variable Quiz

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 37—Linear Transformations & Combining Random Variables—CED 4.9

Learning Objectives—Students will learn...

- the effect of a linear transformation of a random variable on the mean and standard deviation.
- how to find the mean and standard deviation of the combination of two or more random variables.

Lesson suggestions

- I find the best approach for this lesson is to pick one (or two) strong anchor examples that show all the rules, work through it at a relatively fast pace, and then have students dive into some guided practice. The cereal problem in *SMW* is a great option.
- On the Desmos below, I use a quiz score to show linear transformations. First take the quiz scores (20 points possible) and multiply by 5 to turn the scores into a 100 point test. Then add 3 points to the test scores for extra credit!
- The second example on the Desmos Activity uses hamster weights, converts ounces to grams, adds the weight of a box, adds more hamsters, etc. ...
- After a guided practice example, it is valuable to have a conversation about the difference between $X+X \neq 2X$. You can read more about this in the *Teacher Resource Guide* (highly recommended). Here's one example for this discussion:
- Suppose you're playing Monopoly and you lost one of the two dice. Your friend says, "We'll just roll one die and double that number!" Is that the same thing as rolling two dice? Why or why not?
- Discussing these differences is helpful.
    - Evens vs. not evens
    - Uniform vs. symmetrical
    - The difference between a random event added to another random event vs. a random event doubled, etc...
- Doing a normal calculation with a combination of two random variables is the high-water mark of theoretical probability in our course. This is the second day of this lesson. Here is a Peardeck made by myself and my co-teacher Tiffini. It starts with a practice free response (2008 #3) and then moves to golf scores (it is written for a 90-minute block).
- The two golfers have each their own mean and standard deviation. As golfers are wont to do, they compare their scores, so we are interested in their difference. The problem leads first through using RandNormal to simulate 100 games and then the theoretical probability calculation.

Textbook suggestions

- Chapter 15 #29-32, 39, 40-49

Additional resources

AP Classroom Daily Videos

- 4.9—Transformations of random variables
- 4.9—Combinations of random variables

Khan Academy practice

- Transforming Random Variables
- Combining Random Variables
- Combining Normal Random Variables

Desmos

- All the rules, with hamsters!

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 38—Binomial and Geometric Probabilities—CED 4.10 & 4.11 & 4.12

Learning Objectives—Students will learn...

- how to calculate a binomial probability.
- how to calculate a geometric probability.
- the required conditions for a binomial and geometric setting.

Lesson suggestions

- I enjoy using basketball shooting to begin this lesson. Pick a local player and find their shooting percentage. Using independence rules, students can easily calculate a few geometric probabilities on their own.
- Moving to the binomial distribution, I usually have the player shooting 8 shots. Students calculate the probability of making all the shots or missing all the shots. Then we discuss the probability of making exactly 5 of 8. This leads to the development of the formula and (quickly!) to the use of the binomial function on their calculator. We use the calculator to fill in the complete probability model.
- Many of my students have actually never (or only minimally) studied combinations. I am not going to take class time to teach this topic. Instead, I'm going to remind students that they can use their formula sheet to write what their calculator is doing. And that if they don't understand every bit of that, that is OK.
- I don't show my students how to use geometric pdf or cdf on their calculator. I prefer to have them calculate these "by-hand". I think this is a case (for small n) of using a sledgehammer to kill an ant!
- For this first day, I keep drawing probability models (tables) for binomial problems and have them add pdf's by-hand. I wait until later to show/discuss the cdf function.
- While the binomial formula does appear on the AP exam in the form of multiple-choice questions, you should realize that a student can easily receive full credit on a free response problem with strong communication. Simply write a title of "Binomial" and then specify n, p, and the value(s) being calculated. No formula required for full credit.
- Speaking of this formula, students will see multiple-choice items like these on the AP Exam:

  Christmas tree lights can be beautiful. But sometimes they don't work. Suppose that in a string of lights each bulb has a 3% chance of failure. You can assume that each bulb is independent of the next.

  What is the probability that in a string of 50 lights, exactly four of the bulbs fail?

  a) $(0.03)^{50}$
  b) $(0.03)^{4}$
  c) $1-(1-0.03)^{50}$
  d) $(0.97)^{49}(0.03)^{1}$
  e) $\binom{50}{4}(0.03)^{4}(0.97)^{46}$

- Now if you read the question thoroughly, you probably think I have a typo and that I neglected to finish writing the stem of the question. But no! I give my students this problem, as is, and then I ask them to do this:
- Each of the answer options can be correct, *if* you finish writing the question in the proper way. Please figure out all five different endings that make options a-e the correct answer!
- When I present this, students are flabbergasted! You want us to write an AP question?!?! But after their initial shock, most of them do a fine job of it.
- And here's the magic of this exercise: you and I look at those 5 answer options and see 5 different structures. Students look and see chaos! But having them write the rest of the stem forces them to see each answer option for the structure that it represents. I highly recommend this activity as a tool for improving multiple-choice testing performance.

Textbook suggestions

- Chapter 16 #1, 2, 15-18

Additional resources

AP Classroom Daily Videos

- 4.10—Intro to binomials
- 4.12—Probabilities and parameters for geometric, part 1
- 4.12—Probabilities and parameters for geometric, part 2

Khan Academy practice

- Identifying Binomial Variables
- Binomial vs Geometric
- Binomial Probability Formula
- Calculating Binomial Probability
- Geometric Probability

Desmos

- Binomial and Geometric Variables

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 39—Mean & Standard Deviation of Binomial & Geometric Variables—CED 4.11 & 4.12

Learning Objectives—Students will learn...

- how to calculate and interpret the mean and standard deviation of binomial variable.
- how to calculate and interpret the mean and standard deviation of geometric variable.

Lesson suggestions

- This lesson builds naturally from the last. Students can begin class with a binomial example and calculate a few probabilities.
- The formula for a binomial mean is quite intuitive, especially if the numbers are simple (use, say, $n = 10$ and $p = 0.70$). Encourage students to consider what the average is for the warm-up problem.
- Lean into the formula sheet. The new version (2019—CED) neatly organizes the binomial and geometric formulas. Encourage clear communication and interpreting the values in context.
- Students have a funny quirk—they know the mean has the same units as the data, but they aren't quite sure about standard deviations. Make sure students are using units on both values!
- Be aware that the geometric standard deviation was added to the course with the CED in 2019. There are supplemental materials for SMW at this site that include this topic.
- Pacing note: These two lessons may take 3 days.
- Song choice: *One Way or Another*, by Blondie (only two options!). But there are plenty of songs in the world with the number two—I'm sure you might find your own favorite!

Textbook suggestions

- Chapter 16 #19-28
    - Tip: Add geometric sd to problems like #19 & #20.

Additional resources

AP Classroom Daily Videos

- 4.11—Parameters for binomials
- 4.12—Probabilities and parameters for geometric, part 1
- 4.12—Probabilities and parameters for geometric, part 2

Khan Academy practice

- Binomial Mean & SD
- Geometric Mean & SD

Desmos

- Binomial & Geometric Mean & SD
- The 12 days of Christmas problems
    - Variations on this worksheet have been around for many, many years. It is a good review of all sorts of probability problems.

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Free Response Problems—Random variables, simulations, binomial/geometric

- 1998 #6—pearls—old problem, but what a cool simulation!
- 2015 #3—ATM's. Part (d) is such a clever expected value twist.
- 2002 #3—Relay team, combining random variables.
- 2006B #3—binomial and normal with golf balls.
- 2011B #3—Flight upgrades, binomial and geometric.

Lesson 40—Simulations—CED 4.2

Learning Objectives—Students will learn...

- how to read and interpret the results of a simulation.
- how to construct, run, and interpret a simulation.

Lesson suggestions (preamble)

- You may have noticed that I skipped this chapter. Using a random digit table to select a random sample has a long history in AP Stats. One I choose to ignore. I just don't find using a clunky random digit tables to take a SRS to be awesome, especially considering how much technology we all have.
- Let me be clear: random digit tables are in the CED. We don't get to skip them.
- Let me expound: I value teaching simulations as a way of helping students understand probability.
- And finally (before I get to the meat of this lesson), let me confess: I don't use simulations as much as I want to. But I keep trying.

Lesson suggestions

- Depending on my mood and the school calendar, I like to run simulations somewhere during the beginning of probability. It gives students a good feel for the law of large numbers.
- I like to use examples that we will never be able calculate theoretically. For example, "What will be the longest streak, on average, of heads or tails, out of 20 flips?". This one is fun to have students write down 20 coin flips, pretending to be a coin. They will have shorter streaks than a real coin.
- Random.org is fantastic. [I hinted in the preamble that I don't love random digit tables. I tend to demonstrate once and then use them only on multiple choice items. Your mileage may vary.] For the coin streaks above, random.org is just so fast.
- I like using a variety of random creators. Dice, spinners, calculators, tables, websites, etc... I think this strengthens students' understanding.
- I use this 4-step simulation method:
    - Assign your random digits to match the problem.
    - Figure out what it means to run 1 trial. Run it and think about what it means.
    - Run many, many trials.
    - Write a conclusion, answering the question that was asked.
- There was a period of time when there were more simulations on the AP Exam. Here's a (partial) list:
    - 2009 #6 Simulating skewness
    - 2010 #6 Simulated ranks
    - 2009B #5 Bottle filling
    - 2013 #5c Difference of two proportions
    - 2013Secure #6cd Power and p-value simulation
    - 2015Secure #6e Simulated standard deviation ratios
- Perhaps this trend will be resurrected? I hope so.

- Binomial and geometric settings can be simulated. That is another place where I try to incorporate simulations.
- How many days you spend on simulations is very flexible. It takes two days to teach the 4-step simulation process. As I trust I've made clear by now, I think more is more. I start the year with a simulation (Smelling Parkinson's). I also try to incorporate simulations into 2-sample inference.

Textbook suggestions

- Chapter 10 #11, 12, 15, 17, 25-28; Chapter 16 #5, 6

Additional resources

AP Classroom Daily Videos

- 4.2—Intro to simulations
- 4.2—Probability in simulations

Khan Academy practice

- Interpreting Simulations
- Randomness, Probability, Simulation Unit

Desmos

- Simulations
- More Simulations
- Interpreting results of simulations

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 41—Sampling Variability, Bias, and Variability—CED 5.1, 5.2, 5.4

Learning Objectives—Students will learn…

- how to identify if a statistic is biased or unbiased.
- how sample size affects the variability of a statistic.
- that a statistic is a point estimate that will vary upon repeated sampling.

Lesson suggestions

- This is a big topic! You can spend a lot of time discussing when statistics are biased and how they vary. I have tried various ideas for introducing this idea in just one class period.
- Recently, I flipped my classroom. I made a short video on sampling variability and had students watch it. I also assigned normal probability review problems outside of class. The Desmos Activity below can be used for normal review.
- The German Tank Activity has been used for decades for this lesson. Here is one example from DeAnna MacDonald.
- I found success in discussing sampling variability with my students, instead of spending most of the period on such a long activity. I talk about the German Tank problem in context, but only to point out that the sample max is a biased estimator of the population max.
- Students intuitively understand that larger samples will have less sampling variability. The key is point out that this isn't bias—it's an increase/decrease in variability.
- That said, I think Jeff Dodds has written some fantastic Khan Academy practice problems for this topic. I have relied on them heavily the last few years. The link is below.

Textbook suggestions

- Chapter 17 #9, 10

Additional resources

AP Classroom Daily Videos

- Sampling Variability—5.1
- Normal Review, Part 1—3.A
- Normal Review, Part 2—3.A
- Normal Review, Part 3—3.C
- Un/biased Statistics—5.4

Khan Academy practice

- Biased and Unbiased Estimators (highly recommended!)

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 42—T—CED 5.3 & 5.7

Learning Objectives—Students will learn...

- the definition of a sampling distribution.
- how to describe the center, spread, and shape of a sampling distribution of a sample mean.
- what the Central Limit Theorem says about the shape of the sampling distribution of a sample mean.
- how to calculate probability statements about sample means.

Lesson suggestions

- I am very proud of the following resource. With the help of my friends Wes B. and Jeff E., I have created spreadsheets that make a real-world example for teaching this lesson. You can see the latest version here (also in the Desmos Activity below). Jeff is really responsible for improving the layout of this spreadsheet and making the whole process go faster.
- Here's the idea. Take a nice, big, real world dataset. Using a nifty spreadsheet command (vlookup), have students take random samples from this dataset. Then have them compare their sample means, proportions, etc... If you want to speed up the activity, you show students how to use "=RANDBETWEEN" and they will find their random samples more quickly.
- StatCrunch has a sampling distribution applet that will let you simulate sampling distributions from a dataset, instead of just from a random shape. You can use this applet to demonstrate further the center, spread, and shape of the sampling distribution using this applet.
- Lean heavily into the AP formula sheet. It has the mean and standard deviation for the sample mean.
- Connect the three conditions appropriately. Random samples are about center, independence is about the spread, and sample size is about the shape.
- $n > 30$ isn't a real rule. However, it is the rule that the AP exam accepts and acknowledges in the rubrics. Go ahead and use it. It makes life easier for students. But, also, don't be afraid to point out when a practice problem has more nuance in real life.
- This is a two-day lesson. One day to explain the center, shape, and spread. Then a second day to do probability problems.
- I prefer doing means before proportions. But your mileage may vary.

Textbook suggestions

- Chapter 17 #35, 37, 39, 43, 45

Additional resources

AP Classroom Daily Videos

- Intro to Sampling Distributions—5.3
- Center, spread, and shape of a mean—5.7.1
- Probabilities with means—5.7.2

Desmos Activities

- CLT

Khan Academy practice

- Mean and Standard Deviation of a Mean
- Sample Means and the CLT
- Probabilities with Means

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 43—T—CED 5.3 & 5.5

Learning Objectives—Students will learn…

- how to describe the center, spread, and shape of a sampling distribution of a sample proportion.
- what the Central Limit Theorem (for proportions) says about the shape of the sampling distribution of a sample proportion.
- how to calculate probability statements about sample proportions.

Lesson suggestions

- You can use the spreadsheet from the prior lesson to examine the proportion of cars that met the given budget.
- This example lacks real world context, but I made an invisible spinner on a spreadsheet.
- Another option I made was with a physical spinner. I made a pie graph and overlayed it on top of a picture of a delicious pie! The "pie" graph was 42% shaded and 58% not shaded. Students make a spinner by bending one segment of a paperclip and placing a pencil in the middle. Everyone spins, say, 10 or 20 times. We compile a class set of p-hats. Then students guess what p is. I reveal the right answer, and then sampling variability is discussed.
- StatCrunch has a great applet (no log-in required!) that shows a binomial distribution becoming normal. Sometimes a picture is worth a thousand words!
- The > 10 rule, isn't a hard rule. It's a continuum. The applet above demonstrates this. Real life is messy! Sometimes we have to do inference with small samples. And as sample sizes become larger, we are more confident that the shape of the sampling distribution has become normal. Students need to hear both sides of this. On the AP Exam, the conditions will be met. In real life, data are messy.
- Again, lean into the AP formula sheet. And connect checking conditions with center, shape, and spread. And you'll need two days.
- The CED includes 5.6 and 5.8—two sample sampling distributions. I prefer to wait and discuss those topics when I teach those inference topics. This does mean that I can't use all of the PPC problems, but I'm OK with that. I try to spend only about a week on this topic and then dive into inference. I find it more helpful to spiral sampling distribution questions in the inference units than spending too much time on this chapter.

Textbook suggestions

- Chapter 17 #11-23 odd

Additional resources

AP Classroom Daily Videos

- Center, shape, and spread of a proportion—5.5.1
- Probabilities with proportions—5.5.2

Desmos Activities

- CLT for Proportions

Khan Academy practice

- The Normal Condition for Proportions
- Mean and Standard Deviation for Proportions
- Probabilities with Proportions

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 44—One-proportion z-interval—CED 6.2 & 6.3

Learning Objectives—Students will learn...

- how to construct a 1-proportion z confidence interval.
- how to verify that the conditions for a 1-sample proportion confidence interval.
- how to interpret a 1-proportion z-interval in context.

Lesson suggestions

- The fantastically fun way to introduce this topic is to have students flip Hershey's Kisses and find the proportion of Kisses that land on their base. This activity has been around for many, many years. You can find one version in my cloud drive and then there is a Desmos version listed below.
- It is fantastic to bribe students with chocolate and entertain them with Hall & Oates, *Your Kiss is on My List*.
- For the Kiss experiment, I just have students use $z^* = 2$, leaning into their memory about the empirical rule. See the next lesson for refining the value of z^*.
- This lesson has students draw their intervals on the board. Invariably, one is too low and one is too high for every interval to agree. That is great! It demonstrates that sometimes a point estimate will be so extreme that the interval will miss the parameter.
- And speaking of the parameter, I think the best and most honest thing to say about the parameter is that no one knows exactly what it is. Some of my friends have collected point estimates for years and really feel that they know "the true percentage of Hershey's Kisses that will land on their base." Hogwash, I say! No one knows! And that is great. It is the nature of statistics that we try to estimate a parameter that we may never, ever know.

Additional resources

AP Classroom Daily Videos

- Choosing and checking conditions—6.2.1
- Calculating the interval—6.2.2

Khan Academy practice

- Conditions for a 1-prop z-interval
- Finding z^*
- Calculating a 1-prop z-interval

Desmos Activities

- Hershey's Kisses in Desmos
- Intro activity for 2-prop z-interval

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 45—Using Intervals—CED 6.3

Learning Objectives—Students will learn...

- how to interpret a 1-proportion z-interval.
- how to justify a claim using a 1-proportion z-interval.
- how sample size, confidence level, and margin of error are related.

Lesson suggestions

- I photocopy a massive stack of inference templates for students to use. My inference template is simply a sheet of paper divided into 5 sections:
    - Name of inference procedure
    - Null & alternative hypotheses
    - Conditions
    - Math box
    - Conclusion
- I find these templates invaluable for helping students to remember the structure of completing an inference problem for the AP exam.
- So, this is the day we complete our first "blue sheet" (usually copied on blue paper).
- I encourage them to start an example problem, while I provide notes on the board. Hopefully most students use the Kiss problem to stay ahead of me. I show them how to find z^* using both Table B and inverse normal.
- I find students are most successful if they use Table B for z^* and t^*. Most AP problems will use values that can be found on this table.
- As students get used the mechanics, we increase and build the communication about how the interval can be (and not be) interpreted. And how you can use an interval to justify a claim about a particular value.
- We finish the lesson with discussing how the interval changes as the confidence level and sample size change. Problems #7 and #8 in your textbook are perfect for this.

Textbook suggestions

- Chapter 18 #7, 8, 12-17

Additional resources

AP Classroom Daily Videos

- Interpreting an interval and justifying a claim—6.3.1
- Factors that affect margin of error—6.3.2
- The entire process—6.3.3

Khan Academy practice

- Interpreting confidence intervals and levels

Desmos

- Further investigations with intervals

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 46—Solving for sample size—CED 6.2

Learning Objectives—Students will learn...

- how to calculate the sample size necessary to achieve a given margin of error.

Lesson suggestions

- I don't think this lesson requires an entire class period. But there are plenty of deep ideas in the prior lesson that you can wrap up or practice before diving into the algebra inequality of AP Stats.
- Students have a love/hate relationship with this lesson that is directly tied to their algebra skills!
- Telling students that $p = 0.50$ is the safest guess can sound quite mysterious. Pointing out to them that $p(1 - p)$ is just an upside-down parabola that has a vertex at $p = 0.50$ helps remove the mystery for some students.
- I always imagine Jamison yelling at Peter Parker (Spiderman) when I teach this lesson. Jamison wants a given margin of error; Spiderman has to figure out the sample size. YouTube clips of Jamison yelling are easy to find and a nice hook to your lesson.
- Notice that the AP exam multiple-choice questions don't ask that students solve for an exact sample size. They ask, "...which of the given sample sizes will result a margin of error that will satisfy the requirement?". I lean into using multiple-choice assessment items for this topic.
- Make sure that students *always* round up! It is best if your first example has a decimal that is between 0.01 and 0.49 to demonstrate that you're not using normal rounding rules—you're taking that part that is less than one and making it into a whole person.

Textbook suggestions

- Chapter 18 #30, 31

Additional resources

AP Classroom Daily Videos

- Margin of error and sample size—6.2.3

Khan Academy practice

- Sample size and margin of error

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 47—1-proportion z-test—CED 6.4-6.6

Learning Objectives—Students will learn...

- how to write null and alternative hypotheses for a 1-proportion z-test.
- how to verify that the conditions for inference are satisfied for a 1-proportion z-test.
- how to calculate and interpret a test statistic and a *p*-value for a 1-proportion z-test.
- how to write a conclusion for a 1-proportion z-test.

Lesson suggestions

- This is a 2-day lesson. The logic of hypothesis testing takes a while to sink in.
- Encourage students that the vocabulary and ideas that they are about to learn will be repeated over and over again for the rest of the semester. In the language of my classroom, we learn 13 "blue sheets"—5 intervals and 8 tests.
- To start with a clear example, I choose a one-sided test and *p*-value that is small. The textbook has plenty of great exercises you could consider for a first example.
- Some teachers like to start this topic with an inflatable globe. Students toss the globe around the room and report if their index finger landed on water or land. After you collect a decent sample size, you compare this value to whatever value for the percentage of water on earth your students believe.
- You should be able to guide students through one example and then have students finish a second example with guidance.
- On the second day, I introduce a big *p*-value and fail to reject.
- I also love Roxy Peck's card trick. I wrote about it here. ***Highly*** recommended. Students talk about this for the rest of the year.
- I wait a few days before introducing two-sided tests. Usually until after our first test that covers both 1-prop intervals and tests.
- I also have not unveiled the test menu in their calculator. Yet. After the aforementioned first inference test, I release the calculator's power for these problems.

Textbook suggestions

- Chapter 19 #23-30

Additional resources

AP Classroom Daily Videos

- Introduction to testing—6.1
- Writing hypotheses—6.4.1
- Checking conditions—6.4.2
- Calculating the test statistic and *p*-value—6.5.1
- Interpreting the *p*-value—6.5.2
- Writing the conclusion—6.6.1
- The whole process—6.6.2

Khan Academy practice

- Writing hypotheses
- Checking conditions
- Calculating the test statistic
- Calculating the *p*-value
- Writing the conclusion

Desmos Activities

- Introduction to 1-proportion z-test
- A second day of practice

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 48—Errors, Power, & Significance Levels—CED 6.7

Learning Objectives—Students will learn...

- how to describe a Type I error, Type II error, and power in context.
- the consequences of making an error in a significance test.
- that significance level and Type I error are the same probability and have related consequences.
- that Type II error is the complement of power.
- how effect size, significance level, sample size, and standard error affect the probability of a Type II error (and power).

Lesson suggestions

- The longer I teach, the more I appreciate spreading out this lesson throughout teaching inference. I'm currently a fan of using 3 different days spread out through units 5 & 6.
- Most recently, I introduced only Type I and Type II errors first. We interpreted these errors in contexts and discussed their consequences. That was a nice start. We also discussed two-sided tests.
- The next step was not until I taught another piece of inference. Then we discussed significance level, its connection to Type I error, and then its effect on Type II error.
- Finally, I added power.
- It takes a while for the logic of inference to sink in. Spreading out the ideas of this unit give students a better opportunity to understand the subtle ideas. It also gives everyone a break from teaching/learning yet another inference procedure.
- When your students (and you!) are really ready for a challenge, try the Power Dominoes listed below. Dave Bock made a number of statements that students who really understand these concepts should be able to tackle. My friends Corey and DeAnna and I turned these statements into a series of connected dominoes. Students will find this challenging! I have a PDF of this activity in my cloud drive. I use the digits 8675309 on the cards so that I can quickly check their answers. When students are getting stuck on the activity, I just cue up the song with the answer key! (Jenny/867-5309)

Textbook suggestions

- Chapter 20 #1-11 odd, 23-31 odd

Additional resources

AP Classroom Daily Videos

- Type I and II errors—6.7.1
- Power and factors that affect errors—6.7.2

Khan Academy practice

- Note: These exercises use proportions and means
- Type I & II errors
- Error probabilities & power

Desmos Activities

- Introduction to errors
- More on errors (and practice picking the right procedure)
- Power
- Power dominos

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 49—Two Proportion z Inference—CED 6.8-6.11

Learning Objectives—Students will learn...

- how to construct and interpret a 2-proportion z-interval.
- how to justify a claim using a 2-proportion z-interval.
- how to conduct and interpret a 2-proportion z-test.
- how to check the conditions for 2-proportion z-inference.
- how to interpret a p-value for a 2-proportion z-test.

Lesson suggestions

- I skip this lesson and teach it after I've covered all of Unit 6 (t for means). Your mileage may vary!
- Note that the CED recommends 16-18 days for inference for proportions (SMW Chapters 18-21, CED Unit 6). That's a lot of days! We are laying an important foundation. But once that foundation is established, it allows us to teach the rest of the course at a faster and faster pace.
- This lesson takes two days.
- It is fun to collect a 2-way table from students at the beginning of class. I will (with prior permission) send students to the next door classrooms and ask a quick yes/no question. You can compare gender, upper/lower classmen, etc... If you survey your own class plus a few others, you will meet the large sample size requirement. Use a topic that is current at your school that very week.
- Others may disagree, but I don't find calculating this standard error by hand to be clarifying. We start with a test. We write hypotheses, check conditions (more on that below), and then let technology do its thing. We write down the results and students write the conclusion.
- Finally, we discuss what standard error our technology used to find the z-score and p-value. But we don't bother to do the arithmetic.
- After we've written the test conclusion, we add a 95% confidence interval, interpret it, discuss its standard error, and finish by noting whether or not zero is in the interval.

<u>To pool or not to pool?</u>

It seems teachers have trouble keeping these choices clear, so let me be extremely specific.

- For a 2-proportion z-interval, we use the standard error with each p-hat. The combined (pooled) p is not used.
- For a 2-proportion z-test, we are assuming that the two proportions are equal. Since this is our assumption, we need a value that represents our best estimate of what that proportion might be: the combined-p. We use this combined-p *<u>both</u>* to check conditions and to calculate the standard error.
- Note that the formula for the combined-p is on the AP formula sheet.
- Also note that in books published prior to the CED, the conditions check is not accurate. You need to make sure to guide students to use the combined-p when checking conditions for a test (not the individual sample proportions).
- Finally, in the next unit for means, you never pool. This is an unwarranted assumption that we don't need for anything in AP Stats.

Textbook suggestions

- Chapter 21 #11, 19, 21, 23, 33

Additional resources

AP Classroom Daily Videos

- Identifying and checking conditions for a 2-prop interval—6.8.1
- Calculating a 2-prop interval
- Interpreting and using a 2-prop interval—6.9.1
- 2-prop z-interval—the whole process—6.9.2
- Hypotheses for a 2-prop test—6.10.1
- Identifying and checking conditions for a 2-prop test—6.10.1
- Calculations for a 2-prop test—6.11.1
- Interpreting the p-value and conclusion for a 2-prop test—6.11.2
- The entire 2-prop z-test process—6.11.3

Khan Academy practice

- Conditions for 2-props
- 2-prop intervals
- 2-prop hypotheses
- The test statistic for 2-prop
- P-value for 2-props
- Conclusions for 2-prop z-tests

Desmos Activity

- 2-props in Desmos

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 50—1-mean t-inference—CED 7.2-7.5

Learning Objectives—Students will learn...

- how to write hypotheses for 1-mean t inference.
- how to check the conditions for 1-mean t inference.
- how to construct a 1-mean t-interval.
- how to carry out a 1-mean t-test.
- how to interpret the results of 1-mean t-inference.
- the characteristics of the t-distribution.

Lesson suggestions

- At this point, the logic of confidence intervals and hypothesis testing should be fairly strong in the minds of your students. This lesson will take a few days—but it will proceed much quicker than the introduction to inference with proportions.
- Make sure to tell the story of William Gosset and the invention of the t-distribution. Statistics owes a debt of gratitude to the "student" whose beer taste-testing led to this important discovery. The ASA has this PDF about Gosset, which you can add to the standard Wikipedia article.
- I suggest starting with a small sample t-test (I often just use one in the textbook). Present the data, make a dotplot, and find the summary statistics. Students know they need sigma to find the standard deviation of the sampling distribution. Ask them, "What's sigma?". Take *lots* of wrong answers! Once they realize the dilemma, someone will suggest that instead of sigma, we will have to use the sample standard deviation. This leads to the story of Gosset. We have a small sample, we have an estimate of sigma that varies too much, etc...
- I have fully unveiled the calculator functions by this point, so while we draw a curve, label it, etc... I do not do any "by-hand" calculation. Our technology gives us our p-value. Where did it come from? I point to the graph and say "t-cdf", but we don't bother to add extra steps. We let technology do its thing. And once students have a p-value, they can write the conclusion. I just keep emphasizing that this is about a *mean*. We're leaving proportions behind!
- After we run a full hypothesis test, we add a confidence interval to the problem. We use Table B to find the t^* and the rest is easy! We lean into the best part of a table: we can see and compare lots of values all at once. We look at our old friend, 1.96, and note how many more standard errors we need, depending on sample size.
- Finally, we note how the interval and test agree (reject if the parameter is not in the interval, etc...).
- That takes a full class period. The next day, we repeat, with two main differences.
- Difference #1 is that students are working much more independently and are running the procedures for themselves.
- Difference #2 is that on day two I use a problem with only the summary stats and a sample size bigger than 30. This invites students to recall what the CLT has to say regarding the normality condition.
- When the sample size is bigger than 30, this leaves t^* shrouded in a bit of mystery. My preference is to leave t^* as a symbol and to *never* bother figuring it out precisely. Yes, I do

think this is a safe practice for the AP exam. The exam does not require formulas to be filled in completely and it usually uses sample sizes that are on Table B.
- But more importantly, sometimes less is more. Yes, we could teach inverse-t (and t-cdf). But, frankly, no one cares that much. We can look at Table B and see approximately what t^* is. But what do we really care about? I'll wait, you decide.....
- Note that the CED does ***not*** include solving for sample size for a given margin of error for a mean.

Textbook suggestions

- Chapter 22 #7, 13, 17, 31, 39, 47

Additional resources

AP Classroom Daily Videos

- Introduction Video—7.1
- z^*? No! t^*—7.2.1
- Identifying and checking conditions for a t-interval—7.2.2
- Calculating a t-interval—7.2.3
- Interpreting a t-interval and justifying claims—7.3.1
- Interpreting level and factors that affect the margin of error—7.3.2
- The whole process for a t-interval—7.3.3
- Hypotheses for a t-test—7.4.1
- Identifying a t-test and checking conditions—7.4.2
- Calculating a t-test—7.5.1
- Interpreting a p-value and writing a conclusion for a t-test—7.5.2
- The entire t-test process—7.5.3

Khan Academy practice

- Conditions for a t-interval
- Finding t^*
- Calculating a t-interval
- Writing hypotheses for tests (means and props)
- Writing hypotheses for a t-test
- Conditions for a t-test
- Calculating the test statistic for a t-test
- Calculating the p-value for a t-test
- t-test conclusions

Desmos Activities

- 1-mean t-interval
- 1-mean t-test
- Inference practice and review (including props)

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like! This exit ticket contains extra questions, given the multiple-day nature of this lesson.

Lesson 51—Two sample inference for the difference of two means—CED 7.6-7.9

Learning Objectives—Students will learn...

- how to check the conditions for inference for the difference of two means.
- how calculate a t-confidence interval for the difference of two means.
- how interpret a t-confidence interval and justify a claim for the difference of two means.
- how to write hypotheses for a t-test for the difference of two means.
- how to write a conclusion for a t-test for the difference of two means.

Lesson suggestions

- This lesson has one new idea (the standard error) and the rest is all familiar. Push your students to do the lesson without help! And without looking at their notes.
- Pedagogical note: students like to look at their notes and copy. But this is bad for their learning. We learn and remember more when we force our brain to recall information. The process of recall (bringing information from stored memory to active memory) is the way we form powerful memory pathways. We need to encourage our students to form those pathways by remembering all the steps of inference, not by just copying them repeatedly.
- There's not a lot to say here. Pick a good problem. Or generate class data by asking your students a question. A reaction time website (like this one) can be fun for comparing gender differences in speed.
- You might consider doing the confidence interval first, noting if zero is or is not in the interval, and then performing a test.
- This is where technology needs to do its thing. Don't worry about df and t*. We are thankful that smart people figured this out. Let's act like practicing statisticians and interpret results. This isn't an algebra class, nor is it a (painful) lesson in using the order of operations.
- Worried about the formulas? Start writing/using multiple choice items! The goal isn't to calculate this beast. The goal is to be able to use the AP formula sheet to recognize the right formula for the right situation. Also, see my note in Lesson 53 regarding scoring on inference problems.

Textbook suggestions

- Chapter 23 #9-17 odd, 21, 23, 33

Additional resources

AP Classroom Daily Videos

- Identifying and checking conditions for a t-interval for the difference of 2 means—7.6.1
- Calculating a 2-mean t-interval—7.6.2
- Interpret a confidence interval and justify a claim for the difference of two means—7.7.1
- The whole 2-mean t-interval process—7.7.2
- Writing hypotheses for a 2-mean t-test—7.8.1
- Identifying a 2-mean t-test and checking its conditions—7.8.2
- Calculations for a 2-mean t-test—7.9.1

- Interpreting the p-value and writing a conclusion for a 2-mean t-test—7.9.2
- The entire 2-mean t-test process—7.9.3

Khan Academy practice

- Conditions for a 2-mean t-interval
- Your calculator and the 2-mean t-interval
- Writing hypotheses for the difference of two means
- The test statistic for a 2-mean t-test
- The p-value for a 2-mean t-test
- Conclusions for the 2-mean t-test

Desmos Activities

- 2-mean t-inference

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like! This exit ticket contains extra questions, given the multiple-day nature of this lesson.

Lesson 52—Matched pairs t-inference—CED 7.2-7.5

Learning Objectives—Students will learn...

- how to identify a matched pairs problem for t-inference (vs. 2-sample).
- how to write hypotheses for a matched pairs t-test.
- how to check conditions for matched pairs t-inference.
- how to calculate a confidence interval for matched pairs t-inference.
- how to interpret a confidence interval matched pairs t-inference and how to use the interval to justify a claim.
- how to carry out a t-test for matched pairs.
- how to interpret a t-test for matched pairs.

Lesson suggestions

- I have detailed two matched pairs experiments you can have students complete. Tracing a poop emoji (dominant hand vs. non-dominant hand) was described in Lesson 28 [insert link] and snapping speed (again, dom. vs. non-dom.) is contained in Lesson 27 [insert link]. Google forms have made my life organized for activities like these. The data that was inputted in October is waiting in my Google Drive for this lesson.
- Spend some time discussing why individual variance is not what we want to measure. We want to measure *the difference* in speed (for either of the two activities above). Students will realize that a null hypothesis of zero difference makes sense.
- Once you have laid this groundwork, students can finish the rest. They just need two things to guide them:
    - Reminders to only use the difference list.
    - Add a sub-D to their notation. Mu-sub-D, x-bar-sub-D, etc... This is about the mean difference (not the difference of means).
- Students will want to use two means—don't let them!
- Encourage students that this is really a 1-sample procedure, even though we started with two lists.
- This topic hides as a sub-topic in the CED. But it is there. It is part of 1-sample t-inference in sections 7.2 through 7.5.
- As you're teaching this unit, don't forget to spiral in questions about power, errors, changes in sample size, etc...

Textbook suggestions

- Chapter 24 #3, 7, 11, 13, 23, 25, 27, 30

Additional resources

AP Classroom Daily Videos

- Matched pairs vs. 2-mean—7.10.1
- Picking the right procedure—7.10.2

Desmos Activities

- Matched pairs t

AP Free Response Questions for means

- 2013 #1—crows—1-mean t-interval
- 2009B #5—bottle filling—1-mean t test and simulation question
- 2009 #6—measuring skewness—don't miss this *great* investigative task!
- 2000 #4—baby walkers—2 mean t-test
- 2009 #4—ambulance response times—confidence interval for 2-means
- 2011 #4—cholesterol drugs—2-mean t-test
- 2007 #4—e. coli in beef—matched pairs t-test
- 2014 #5—car prices—matched pairs t-test

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like! This exit ticket contains extra questions, given the multiple-day nature of this lesson.

Lesson 53—Chi-square Goodness of fit test—CED 8.2 & 8.3

Learning Objectives—Students will learn...

- how to write hypotheses for a chi-square goodness of fit test.
- how to check conditions for a chi-square goodness of fit test.
- how to complete a chi-square goodness of fit test.
- how to make an appropriate conclusion for a chi-square goodness of fit test.

Lesson suggestions

- Time for Froot Loops! Buy one big box per class and pass them out! I just give one small bowl to each pair and have them count out 50 for their sample.
- I picked 50 on purpose—it's not divisible by six. I want the expected counts to be non-whole numbers. Rounding expected counts wrong—make sure to reinforce this from the start.
- Note this wording in the CED: "...the alternative hypothesis is that at least one of these proportions is not as specified in the null hypothesis." There are lots of ways to word the hypotheses on these questions. But given how specific the CED is for this particular alternative hypothesis, that is what I'm going to model and encourage. Are there other ways to write the alternative that will receive full credit in future scoring guidelines? Almost certainly. But why not pick the safest path?

Important topic: Scoring Guidelines for Inference Procedures

- Speaking of safe path, let's take a minute to discuss the scoring guidelines for inference. The guidelines (without fail) say that the proper procedure must be identified "...by name or by formula". That means that if students write the correct name, they don't have to write the formula.
- Now that doesn't mean students shouldn't know all these formulas. For one, they need them to answer multiple choice questions, where answer options are often written as formulas. And for two, writing the formula does communicate better understanding. And if the rubric has four parts, it can be one consideration that leads to a score being rounded up (e.g., 2.5 rounds to a 3 instead of down to a 2).
- However, some students will struggle with the formulas (especially the 2-sample formulas) and writing them down incorrectly will lower their score.
- Here's the advice I give my students: write the formula, with numbers plugged in, if you know you can do it right. <u>Always</u> make sure to write the name of the test/interval.
- DON'T write the formula with symbols. You don't get any extra points for that and there are too many ways to make a mistake.
- For chi-square, you have to show the expected counts and the formula is directly on the formula sheet, so make sure to do <u>both</u>, name and formula.
- If you don't write the formula, make sure to write down everything your calculator gave you. Also, make sure to communicate thoroughly in the context and define the parameters. Strong communication can also help your score round up.
- Finally, keep using multiple-choice items with formulas. That will force your students to keep working on them.

Back to Goodness-of-fit

- After running the full test on our Froot Loops, most groups will fail to reject. However, one group usually doesn't (1 out of 20!), so we discuss the Type I error that occurred.
- Before they leave class that day, they start another test, this one with the expected counts distributed according to certain percentages instead of uniform.
- Students find this lesson to be fairly simple! So, start spiraling for the AP test. The next day you could review an old topic for your Do-Now and then practice a bit more.

Textbook suggestions

- Chapter 25 #1-7 odd, 13-15

Additional resources

AP Classroom Daily Videos

- Introductory video—8.1
- Properties of chi-square—8.2.1
- Writing hypotheses for chi-square goodness of fit test—8.2.2
- Identifying a chi-square goodness of fit test and checking its conditions—8.2.3
- Calculating a chi-square goodness of fit test—8.3.1
- Interpreting the p-value and writing the conclusion for chi-square goodness of fit test—8.3.2
- The whole process for chi-square goodness of fit test—8.3.3

Khan Academy practice

- Expected counts for goodness-of-fit
- Conditions for goodness-of-fit
- Calculating goodness-of-fit
- Conclusions for goodness-of-fit

Desmos Activity

- Goodness of fit

AP Free Response

- 2008 #5—Moose
- 2003B #5—skunk spinner

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 54—Chi-square for 2-way tables—CED 8.4-8.6

Learning Objectives—Students will learn...

- how to write hypotheses for chi-square tests for 2-way tables.
- how to check conditions for chi-square tests for 2-way tables.
- how to the calculations for chi-square tests for 2-way tables.
- how to interpret p-values and write conclusions for chi-square tests for 2-way tables.

Lesson suggestions

- It is true that there are two different procedures that exist because there are two different data collection methods that lead to a 2-way table. However, professional statisticians don't really care so much about this difference. Thus, I make the decision to triage this difference as one that I will not allot much class time or student attention to.
- I do point out to students the two different ways we can construct a 2-way table. But then I encourage students to simply...
    - Use the name "chi-square test for 2-way tables".
    - Make sure to write the formula for one chi-square component (see the prior lesson for name *or* formula discussion).
    - Use the wording of the question to write the hypotheses.
- If you're looking for some official support from an AP exam question to support this approach, look no further than 2016 #2 which used the phrasing "association" in a way that is broader than the usual distinction some teachers try to make.
- This is the third time students have examined 2-way tables for independence. They did this back in the very first week of the year and they did this again in the probability unit. Every time we cover this topic, I give students a table that is empty, except for the totals. Then I ask students to fill in the table to make it perfectly independent. This is *not* a tricky formula. All that is required is some 6^{th} grade cross-multiplying! Completing this process (perhaps as a Do-now for this lesson) helps remove the mystery from the chi-square calculation that technology runs without any expected counts provided.
- Don't belabor this test! Once the hypotheses are writing and a matrix of values is in the calculator, the results can be displayed and a conclusion can be written. At the point in the course, given a pair of hypotheses and p-value, students can write a conclusion with no help! And once the calculation in the calculator is complete, writing down the expected matrix and verifying the values are greater than 5 is simple.
- You're near the end of the course, so teaching this topic quickly (just one day to demonstrate and another practice) won't really be quite enough. But it is time to start cumulative review for the exam. So, spend a few days, spiral in some old topics, then revisit. See the Lesson 56 for ideas about graphic organizers and picking the right inference procedure.

Textbook suggestions

- Chapter 25 #6, 9, 25-28, 35

Additional resources

AP Classroom Daily Videos

- Calculating expected counts—8.4
- Hypotheses for 2-way tables—8.5.1
- Checking conditions for 2-way tables—8.5.2
- Calculations for 2-way tables—8.6.1
- P-values and conclusions for 2-way tables—8.6.2
- The whole process for 2-way tables—8.6.3

Khan Academy practice

- Expected counts for 2-way tables
- Calculations in 2-way table tests
- Conclusions for 2-way table tests

Desmos Activity

- 2-way tables
- J. Anker's chi-square lesson

AP Free Response Questions

- 2013 #4—Fruits and veggies
- 2016 #2—Choco-zuties
- 2017 #5—Schizophrenia

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 55—Inference for slope—CED 9.2-9.5

Learning Objectives—Students will learn…

- how to write hypotheses for a t-test for slope.
- how to check conditions for t inference for slope.
- how to calculate a t-test and a t-interval for slope.
- how write conclusions about the slope.

Lesson suggestions

- This is it! The last topic! But before you dive in, review. Grab your favorite regression problem and spend a full day reviewing regression. You might consider using the Super Six, which is located in my cloud drive.
- This topic is rarely tested on the AP exam and students are always supplied with regression output from the computer. Thus, I choose to streamline this lesson by focusing on reading the output and not starting with a dataset.
- If you did a full day of regression review, you could take that problem and build from there.
- Writing the hypotheses is easy and all the math is complete. The biggest catch is checking conditions. Here, again, I choose to streamline the process. I remind students that we want a scattered residual plot, but I only mention normality briefly (IMO, this topic is actually above students' paygrade).
- After we write the test, we add a confidence interval. I prefer to choose a problem with df on Table B.
- I'll say this one last time, it's time to spiral! We spend one (really fast) day on this topic. Then it's time for more review (more Super Six). Then we revisit. This actually leads to stronger learning. Students are activating the recall process, which strengthens memory pathways.

Textbook suggestions

- Chapter 26 #1-11 odd, 14, 17, 19

Additional resources

AP Free Response Questions

- 2005B #5—walking speed—interval
- 2011 #5—windmills—test

AP Classroom Daily Videos

- Introduction to inference for slope—9.1
- The sampling distribution for slope—9.2.1
- Identifying and checking conditions for a confidence interval for slope—9.2.2
- Calculating a confidence interval for slope—9.2.3
- Interpreting a confidence interval for slope and using it to justify a claim—9.3.1
- Confidence interval for slope—the whole process—9.3.2
- Writing hypotheses for slope—9.4.1

- Identifying a test for slope and checking its conditions—9.4.2
- Calculations for a t-test for slope—9.5.1
- Interpreting a p-value and writing a conclusion for the slope—9.5.2
- The whole process—t-test for slope—9.5.3

Khan Academy practice

- Confidence interval for slope
- Test for slope
- Conclusions about slope

Exit ticket: This link will create a Google form in your own Google drive. The form has an exit ticket ready to go. The link creates your own new copy of the form, so feel free to customize as you'd like!

Lesson 56—Name that test!

Learning Objectives—Students will learn…

- how to select the correct inference procedure.

Lesson suggestions

Graphic Organizer

- The resources I have created for this lesson can be found in my cloud drive. Open the Test Prep folder and then the Name that Test folder. I also posted the materials on my blog.
- Marzano's important research into effective instructional strategies showed this: students learn more when create graphic organizers than anything else! Thus, while SMW has a beautiful summary chart of inference procedures both in the text and on the inside-back cover, I strongly recommend that you have your students make their own.
- I cannot over-emphasize this point. With 100% confidence, I would assert that this is the most important lesson of my school year! I usually have students make this organizer at the end of Unit VI and then have students add Unit VII as we complete each chapter. Your mileage may vary.
- Here's how I structure this lesson. I make copies of the chart in the back of the book, except empty. I hide all the textbooks that are normally in the room. Students get out their AP formula sheet and the blank chart.
- I guide them in filling out the top line—1 prop z-inference. I pause and encourage them to fill out every answer ahead of me. Then I give them the rest of the period to finish the chart.

Name that test!

- Each set of Unit Summary exercises have mixed-up inference problems. Give students a list of problem #'s that cover every type of inference they have learned so far. Give them about 3 minutes per problem to name each problem. Then assign as much practice as you'd like; problems that they need to not just name, but finish completely.
- Point out to students that just naming a procedure is harder than doing it. Often their calculator's required inputs will guide them to realizing that they've picked the wrong test/interval. Matched pairs vs. 2-mean is the most tricky.
- After you've completed the text and are reviewing for the exam, make sure to start using scrambled sets of old FRQ's.
- I organized 30 old free response questions into 3 sets of 10. See the cloud drive link at the top of this post.
- I like to liven up this activity by announcing "Name that test!" as if it is a game show. Herb Albert and the Tijuana Brass' "Spanish Flea" is a great musical introduction to any game show!

Textbook suggestions

- Unit V, VI, and VII summaries

Additional resources

AP Classroom Daily Videos

- Picking the right chi-square—8.7
- Reviewing all inference—9.6

Khan Academy practice

- Choosing the right inference procedure

Desmos Activities

- Name that test!